LIE GROUP ANALYSIS OF DOUBLE DIFFUSIVE CONVECTION OVER AN INCLINED SURFACE WITH RADIATION AND HEAT GENERATION

Aasaithambi Thangaraj
Department of Mathematics, Sri Shanmugha College of Engineering and Technology, Pullipalayam, Sankari, Salem-637304, Tamilnadu, India.

ABSTRACT
The investigation of natural convection heat and mass transfer fluid flow past an inclined semi-infinite porous surface with thermal radiation and heat generation is done by Lie group analysis. Scaling symmetries reduce the governing partial differential equations to a system of ordinary differential equations, and numerical solutions for scaling symmetry are obtained. It is found that the fluid's temperature and velocity increase with the heat generation parameter, while its concentration decreases with an increase in the heat generation parameter. Additionally, it is found that the fluid's temperature and concentration decrease with an increase in the porosity parameter, while its velocity increases with the porosity parameter.

Keywords: Lie groups, Natural convection, Porous medium, Radiation, Heat generation.

1 INTRODUCTION
The intriguing applications of studying the mass transfer and spontaneous convection heat phenomena in porous media are drawing more attention. Thermal recovery processes in reservoir engineering, the chemical industry, and the study of the dynamics of a sea's hot and salty springs frequently involve processes involving heat and mass transport in porous media. The coupled thermo-solutal natural convection in porous media is also observed in grain storage, evaporative cooling, solidification, and underground dispersion of chemical wastes and other contaminants. Understanding how heat is produced in fluids is crucial when tackling chemical reaction difficulties. Potential consequences of heat generation could change the temperature distribution and, in turn, the pace at which particles are deposited in semiconductor wafers, electric chips, and nuclear reactors. In this study, a boundary layer problem with heat generation caused by natural convection is treated using symmetry approaches. The primary benefit of these approaches is their ability to effectively solve non-linear differential equations. Any continuous collection of transformations that the differential equations remain invariant under—that is, a symmetry group that maps any solution to another solution—are known as symmetries. Since the symmetry solutions reduce the number of independent variables in the problem, they are very well-liked.

Chen [2] performed an analysis to study the natural convection flow over a permeable inclined surface with variable wall temperature and concentration in 2004. The results show that the velocity is decreased in the presence of a magnetic field. Increasing the angle of inclination decreases the effect of buoyancy force. Heat transfer rate is increased when the Prandtl number is increased, Megahed et. al. [3] conducted the study. Convective heat and mass transfer along a semi-infinite vertical flat plate in the presence of a strong non-uniform magnetic field and the effect of Hall currents are analyzed using the scaling group of transformations; see 2003. Molla et al. [4] investigate the effect of natural convection flow on a sphere with heat generation in

2005.They found that both the velocity and temperature increase significantly when the values of heat generation parameter increases.

Ibrahim et al. [5] investigate the similarity reductions for problems of radiative and magnetic field effects on free convection and mass-transfer flow past a semi-infinite flat plate are investigated, 2005. They obtained new similarity reductions and found an analytical solution for the uniform magnetic field by using Lie group method. They also presented the numerical results for the non-uniform magnetic field. Using the group theory method, Kalpakides and Balassas [6] looked into the free convective boundary layer problem of an electrically conducting fluid on top of an elastic surface.in 2004.. Their results agree with the existing result for the group of scaling symmetry. They found that the numerical solution also does so.

In 1997, Yurusoy and Pakdemirli [7] used Lie group analysis to look into three-dimensional, non-Newtonian fluids with unsteady, laminar boundary layer equations.They assume that the shear stresses are arbitrary functions of the velocity gradients. Using Lie group analysis, they obtained two different reductions to ordinary differential equations. They also studied the effects of a moving surface with vertical suction or injection through the porous surface. The boundary layer equations of a specific non-Newtonian fluid traveling over a stretching sheet were solved exactly in 1999 by Yurusoy and Pakdemirli [8] using the Lie group analysis method. They found that the boundary layer thickness increases when the non-Newtonian behaviour increases. They also compared the results with that for a Newtonian fluid.

So far, no attempt has been made to study the heat and mass transfer in an inclined porous surface with heat generation using Lie groups and hence we study the problem of natural convection heat and mass transfer flow past an inclined porous plate with thermal radiation and heat generation for various parameters using Lie group analysis.

2 MATHEMATICAL ANALYSIS

Consider the heat and mass transfer by natural convection in laminar boundary layer flow of an incompressible viscous heat generating fluid along a semi-infinite inclined porous plate with an acute angle α from the vertical. The thermal radiation and internal heat generation are included. The Roseland approximation is used to implement the thermal radiation. The surface is maintained at a constant temperature T_ω which is higher than the constant temperature T_∞ of the surrounding fluid and the concentration C_ω is greater than the constant concentration C_∞. The fluid properties are assumed to be constant. The governing equations of the mass, momentum, energy and concentration for the steady flow can be written as,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)cos\alpha + g\beta^*(C - C_\infty)cos\alpha - \frac{\upsilon}{k'}u, \tag{2}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_\infty) - \frac{\lambda}{k}\,\frac{\partial q_r}{\partial y}, \tag{3}$$

$$u\,\frac{\partial C}{\partial x} + v\,\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

with the boundary conditions

$$\begin{aligned} &u = v = 0, \qquad && T = T_\omega, \qquad && C = C_\omega \qquad && at\ \ y = 0,\\ &u = 0, && T = T_\infty, && C = C_\infty, && as\ \ y \to \infty, \end{aligned} \tag{5}$$

where u and v are the velocity components; x and y are the space coordinates; T is the temperature; C is the concentration; υ is the kinematic viscosity of the fluid; g is the acceleration due to gravity; β is the coefficient of thermal expansion; β^* is the coefficient of expansion with concentration; λ is the thermal diffusivity; q_r is the local radiative heat flux; k is the thermal conductivity of fluid; ρ is the density of the fluid; c_p is the specific heat of the fluid; D is the diffusion coefficient k' is the permeability of the porous medium; Q is the volumetric rate of heat generation and α is the angle of inclination.
The non-dimensional variables are

$$\overline{x} = \frac{x}{L}, \quad \overline{y} = \frac{y}{L}, \quad \overline{u} = \frac{uL}{\upsilon}, \quad \overline{v} = \frac{vL}{\upsilon}, \quad \theta = \frac{T-T_\infty}{T_\omega - T_\infty}, \quad \phi = \frac{C-C_\infty}{C_\omega - C_\infty}. \qquad (6)$$

where $\overline{u}$ and $\overline{v}$ are the dimensionless velocity components; $\overline{x}$ and $\overline{y}$ are the dimensionless space coordinates; L is the characteristic length; θ is the dimensionless temperature; and ϕ is the dimensionless concentration.
Substituting (6) into equations (1)-(4) and dropping the bars, we obtain,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (7)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\,\theta\,cos\alpha - Gc\,\phi\,cos\alpha - \frac{1}{K}u, \qquad (8)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}(1+4R)\frac{\partial^2 \theta}{\partial y^2} + S\theta, \qquad (9)$$

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}, \qquad (10)$$

with the boundary conditions

$$\begin{aligned} &u = v = 0, \quad &\theta = 1, \quad &\phi = 1 \quad &at\ y = 0, \\ &u = 0, \quad &\theta = 0 \quad &\phi = 0 \quad &as\ y \to \infty, \end{aligned} \qquad (11)$$

where $Gr = \frac{g\beta(T_\omega - T_\infty)L^3}{\upsilon^2}$ is the Grashof number, $Gc = \frac{g\beta(C_\omega - C_\infty)L^3}{\upsilon^2}$ is the solutal Grashof number, $Pr = \frac{\rho c_p \upsilon}{k}$ is the Prandtl number, $Sc = \frac{\upsilon}{D}$ is the Schmidt number, $K = \frac{k'}{L^2}$ is the3 dimensionless permeability parameter, $R = \frac{4\sigma_0 T_\infty^3}{3kk^*}$ is the radiation parameter and $S = \frac{QL^2}{\rho c_p \upsilon}$ is the heat generation parameter.

3 SYMMETRY GROUPS OF EQUATIONS

Bluman and Kumei [1] calculated the symmetry groups of equations (7)-(10) using the classical Lie group approach in 1989. The one-parameter infinitesimal Lie group of transformations leaving (7)-(10) invariant is defined as

$$\begin{aligned} x^* &= x + \epsilon\xi_1(x,y,u,v,\theta,\phi) \\ y^* &= y + \epsilon\xi_2(x,y,u,v,\theta,\phi) \\ u^* &= u + \epsilon\eta_1(x,y,u,v,\theta,\phi) \\ v^* &= v + \epsilon\eta_2(x,y,u,v,\theta,\phi) \\ \theta^* &= \theta + \epsilon\eta_3(x,y,u,v,\theta,\phi). \\ \phi^* &= \phi + \epsilon\eta_4(x,y,u,v,\theta,\phi) \end{aligned} \qquad (12)$$

The corresponding generator X is defined as

$$X = \xi_1\frac{\partial}{\partial x} + \xi_2\frac{\partial}{\partial y} + \eta_1\frac{\partial}{\partial u} + \eta_2\frac{\partial}{\partial v} + \eta_3\frac{\partial}{\partial \theta} \qquad (13)$$

The total derivative operator is

$$D_i = \frac{\partial}{\partial x_i} + u_i^\mu\frac{\partial}{\partial u^\mu} + u_{ij}^\mu\frac{\partial}{\partial u_j^\mu} + \cdots + u_{i,i_1,i_2,\dots,i_n}^\mu\frac{\partial}{\partial u_{i_1,i_2,\dots,i_n}^\mu} \qquad (14)$$

with summation over a repeated index.
The invariance condition in Bluman and kumei [1] becomes

$$\eta_1^{(1)1} + \eta_2^{(1)2} = 0$$
$$\eta^1 u_1 + u\eta_1^{(1)1} + \eta^2 u_2 + v\eta_2^{(1)1} = \eta_{22}^{(2)1} + Gr\eta^3 cos\alpha - \frac{1}{K}\eta^1 \quad (15)$$
$$\eta^1\theta_1 + u\eta_1^{(1)3} + \eta^2\theta_2 + v\eta_2^{(1)3} = \frac{1}{Pr}(1+4R)\eta_{22}^{(2)3} + S\eta^3$$
$$\eta^1\phi_1 + u\eta_1^{(1)4} + \eta^2\phi_2 + v\eta_2^{(1)4} = \frac{1}{Sc}\eta_{22}^{(2)4}$$

where,

$$\eta_1^{(1)1} = D_1\eta^1 - (D_1\xi_1)u_1 - (D_1\xi_2)u_2$$
$$\eta_2^{(1)1} = D_2\eta^1 - (D_2\xi_1)u_1 - (D_2\xi_2)u_2$$
$$\eta_2^{(1)2} = D_2\eta^2 - (D_2\xi_1)v_1 - (D_2\xi_2)v_2$$
$$\eta_1^{(1)3} = D_1\eta^3 - (D_1\xi_1)\theta_1 - (D_1\xi_2)\theta_2$$
$$\eta_1^{(1)4} = D_1\eta^4 - (D_1\xi_1)\phi_1 - (D_1\xi_2)\phi_2 \quad (16)$$
$$\eta_2^{(1)3} = D_2\eta^3 - (D_2\xi_1)\theta_1 - (D_2\xi_2)\theta_2$$
$$\eta_2^{(1)4} = D_2\eta^4 - (D_2\xi_1)\phi_1 - (D_2\xi_2)\phi_2$$
$$\eta_{22}^{(2)1} = D_2\eta_2^{(1)1} - (D_2\xi_1)u_{21} - (D_2\xi_2)u_{22}$$
$$\eta_{22}^{(2)3} = D_2\eta_2^{(1)3} - (D_2\xi_1)\theta_{21} - (D_2\xi_2)\theta_{22}$$
$$\eta_{22}^{(2)4} = D_2\eta_2^{(1)4} - (D_2\xi_1)\phi_{21} - (D_2\xi_2)\phi_{22}$$

Substituting (14) and (16) into (15) and then eliminating v_2, u_{22}, θ_{22} and ϕ_{22} through substitution of (8), (9) and (10), and solving we obtain the infinitesimals as

$$\xi_1 = c_1x - c_2$$
$$\xi_2 = -\alpha(x)$$
$$\eta_1 = c_1u \quad (17)$$
$$\eta_2 = -u\alpha'(x)$$
$$\eta_3 = c_1\theta$$
$$\eta_4 = c_1\phi.$$

Imposing the restrictions from boundaries and from the boundary conditions on the infinitesimals, we obtain the following form for equations (17)

$$\xi_1 = c_1x - c_2$$
$$\xi_2 = 0$$
$$\eta_1 = c_1u \quad (18)$$
$$\eta_2 = 0$$
$$\eta_3 = c_1\theta$$
$$\eta_4 = c_1\phi.$$

where the parameters c_1 represents the scaling transformation and parameter c_2 represents translation in the *x* coordinate.

4 REDUCTION TO ORDINARY DIFFERENTIAL EQUATIONS

In this section, parameter c_1 is taken to be arbitrary and all other parameters are zero in (18).
The characteristic equations are

$$\frac{dx}{x} = \frac{dy}{0} = \frac{du}{u} = \frac{dv}{0} = \frac{d\theta}{\theta} = \frac{d\phi}{\phi} \quad (19)$$

from which the similarity variables, the velocities the temperature and the concentration turn out to be of the form

$$\eta = y\,, \quad u = x\,F_1(\eta)\,, \quad v = F_2(\eta)\,, \quad \theta = x\,F_3(\eta)\,, \quad \phi = x\,F_4(\eta)\,. \qquad (20)$$

Substituting (20) into equations (7)-(10), we finally obtain the system of nonlinear ordinary differential equations

$$\begin{aligned} F_1'' &= F_1^2 + F_2F_1' - GrF_3cos\alpha - GcF_4cos\alpha + \frac{1}{K}F_1 \\ F_2' &= -F_1 \\ F_3'' &= \frac{Pr}{1+4R}(F_2F_3' + F_1\,F_3 - SF_3) \\ F_4'' &= Sc(F_2F_4' + F_1\,F_4). \end{aligned} \qquad (21)$$

The appropriate boundary conditions are expressed as

$$\begin{aligned} &F_1 = F_2 = 0, \quad F_3 = 1, \quad F_4 = 1 \quad \text{at } \eta = 0, \\ &F_1 = 0, \quad F_3 = 0, \quad F_4 = 0 \quad \text{as } \eta \to \infty. \end{aligned} \qquad (22)$$

5 NUMERICAL METHODS FOR SOLUTIONS

A numerical treatment would be more suitable due to the highly nonlinear nature of the equations. A fourth order Runge-Kutta method and shooting techniques are used to numerically solve the system of transformed equations (21) and the boundary conditions (22) with a systematic guessing of $F_1'(0), F_3'(0)$ and $F_4'(0)$. Until the findings reach the required level of precision, which is 10^{-5}, the process is repeated. The MATHEMATICA package is used to write the code, and the results are displayed graphically.

6 RESULTS AND DISCUSSIONS

Numerical solutions are carried out for various values of the Prandtl number, thermal Grashof number, solutal Grashof number, Schmidt number, the porosity parameter and the heat generation parameter. Prandtl number Pr is varied from 0.005 to 2.05, thermal Grashof number Gr from 0.1 to 0.7, solutal Grashof number Gc from 0.1 to 1.0 Schmidt number Sc from 1 to 10, the porosity parameter K from 1 to 5 and the heat generation parameter S from 0.1 to 1 with the angle of inclination α taking the value 45^o. The numerical results are depicted graphically in the form of velocity, temperature and concentration profiles.

The effect of the Prandtl number on the velocity, temperature and concentration distributions is presented in the Figures 1 (a-c). The velocity decreases with increasing Prandtl number, that is, the effect of Prandtl number is to retard the flow, Figure 1(a). The temperature decreases with increase in the Prandtl number, see Figure 1(b). This is in agreement with the physical fact that the thermal boundary layer thickness decreases with increase in the Prandtl number. The reason for such a behaviour is that the high Prandtl number fluid has a relatively low thermal conductivity. This results in the reduction of the thermal boundary layer thickness. When increasing the Prandtl number the concentration distribution increases, Figure 1(c). The effect of the Grashof number on the velocity, temperature and concentration distributions is presented in the Figures 2 (a-c). It is seen from the Figure 2(a) that the velocity increases with the thermal Grashof number because the favourable buoyancy force accelerates the fluid. The temperature and concentration decrease with increasing the thermal Grashof number. Consequently, the thermal and solutal boundary layer thicknesses are reduced.

Figures 3(a-c) deal with the effect of the porosity parameter on the velocity, temperature and concentration distributions for $S = 0.1, Gr = 1, Gc = 0.1, Pr = 0.71$ and $Sc = 1$. The velocity increases with increasing porosity parameter. Temperature and concentration decrease with increasing porosity parameter. The effect of radiation parameter on flow, heat and mass fields is depicted in the Figures 4(a-c). When the radiation parameter is increased the velocity of the fluid and temperature increases. Also it is found that small changes occur in solutal boundary layer when changes made in radiation parameter. Figures 5(a-c) display the velocity, temperature and concentration profiles for different values of the heat generation parameter $S(=0.1, 0.6, 1)$ having $K=5, Gr=1, Gc=0.1, Pr=0.71$ and $Sc=0.1$. It is seen from Figure 5(a) that the velocity profile is influenced considerably and increases when the value of heat generation parameter increases. From Figure 5(b), we observe that when the value of heat generation parameter increases, the temperature distribution also increases along the boundary layer. Also it is observed that the concentration profile decreases significantly when the heat generation parameter increases, Figure 5(c). The effect of inclination of surface on flow, heat and mass fields is depicted in the Figures 6(a-c). When the inclination is increased the velocity of the fluid increases. Also it is found that both thermal and solutal boundary layers are increased with the inclination. The effect of Schmidt number on velocity, temperature and concentration profiles is displayed in Figure 7(a-c). The velocity distribution increases when rising the Schmidt number. Increasing Schmidt number, the thickness of solutal and thermal boundary layer is decreased.

7 CONCLUSIONS

The partial differential equations' symmetries are first determined by Lie group analysis, and then the equations are reduced to ordinary differential equations through the use of scaling symmetries. Both the thermal and concentration boundary layer thicknesses drop when the thermal Grashof number and Schmidt number grow, according to the numerical results, whereas the solutal Grashof number increases both thicknesses. Concentration falls when the heat generation parameter increases, while the fluid's temperature and velocity increase. A higher porosity parameter causes the fluid's velocity to increase while its temperature and concentration fall. As the Prandtl number rises, the temperature falls and the concentration increases. Also it is observed that the velocity and thickness of momentum boundary layer increase with increasing thermal Grashof number and Schmidt number and decrease with increasing Prandtl number and solutal Grashof number.

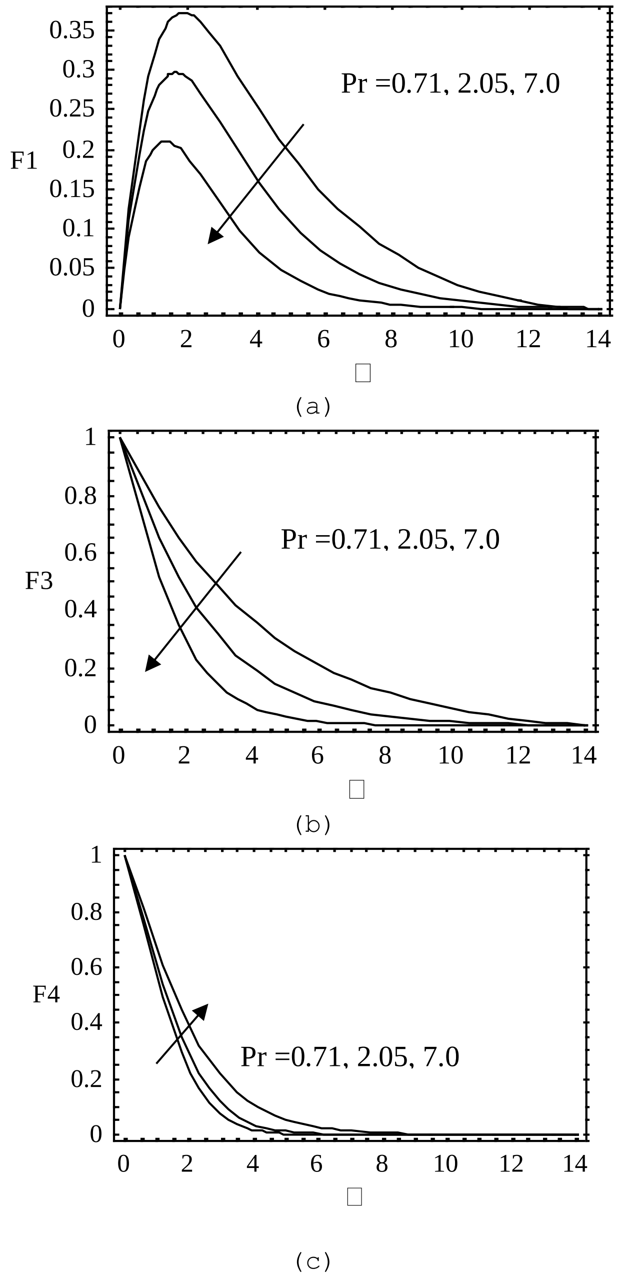


Figure 1 Velocity and temperature profiles for different Pr, Sc=1 Gr=1, Gc=0.1, R =1, S =0.1 and K= 5

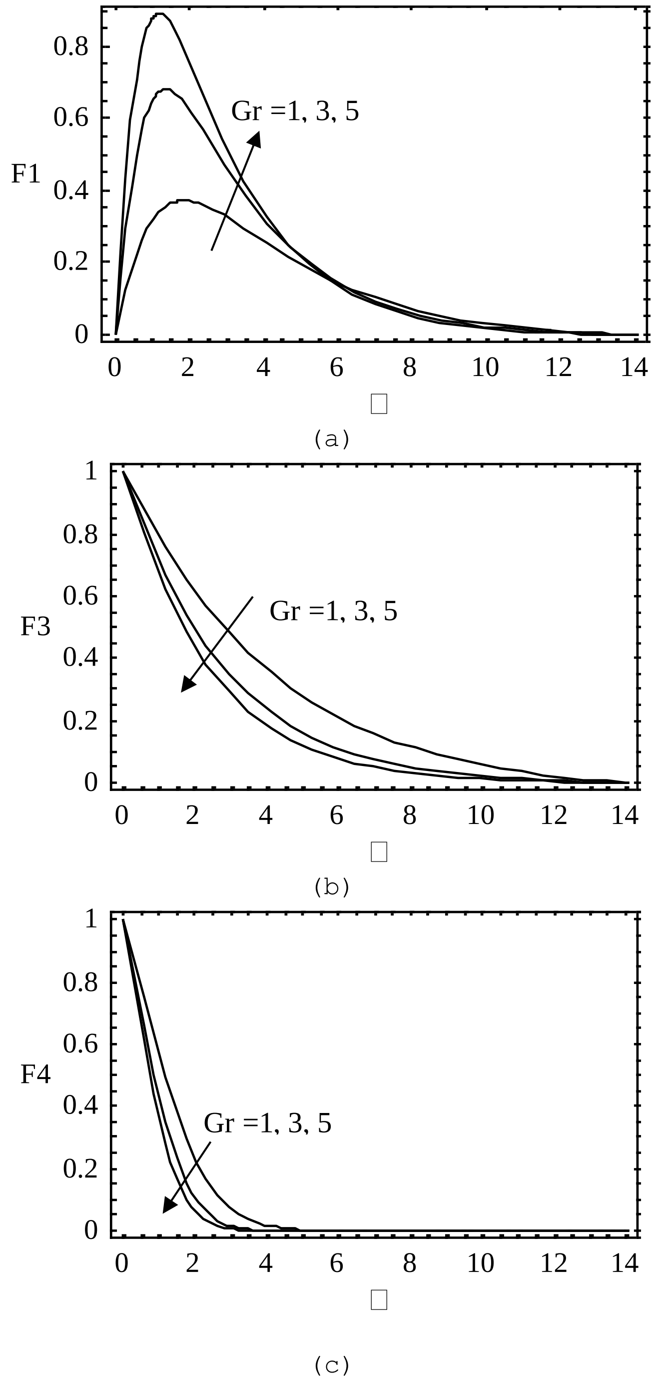


Figure 2 Velocity and temperature profiles for different Gr, Sc=1 Pr=0.71, Gc=0.1, R =1, S =0.1 and K= 5

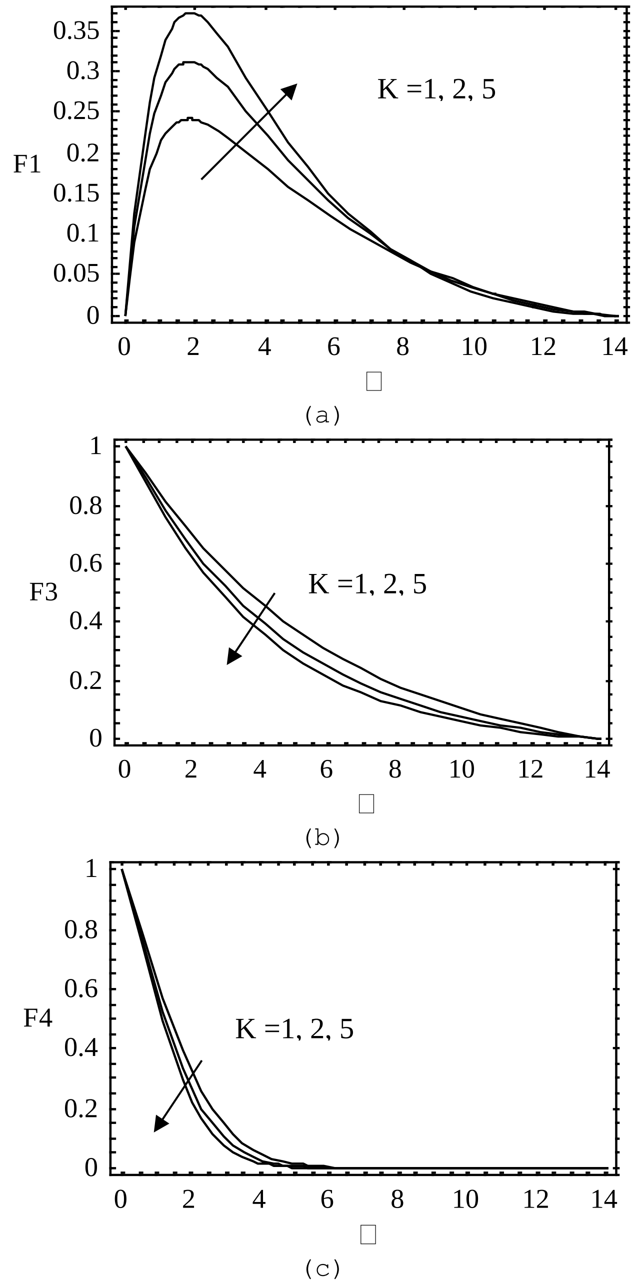


Figure 3 Velocity and temperature profiles for different K, Sc=1 Pr=0.71, Gc=0.1, R =1, S =0.1 and Gr =1

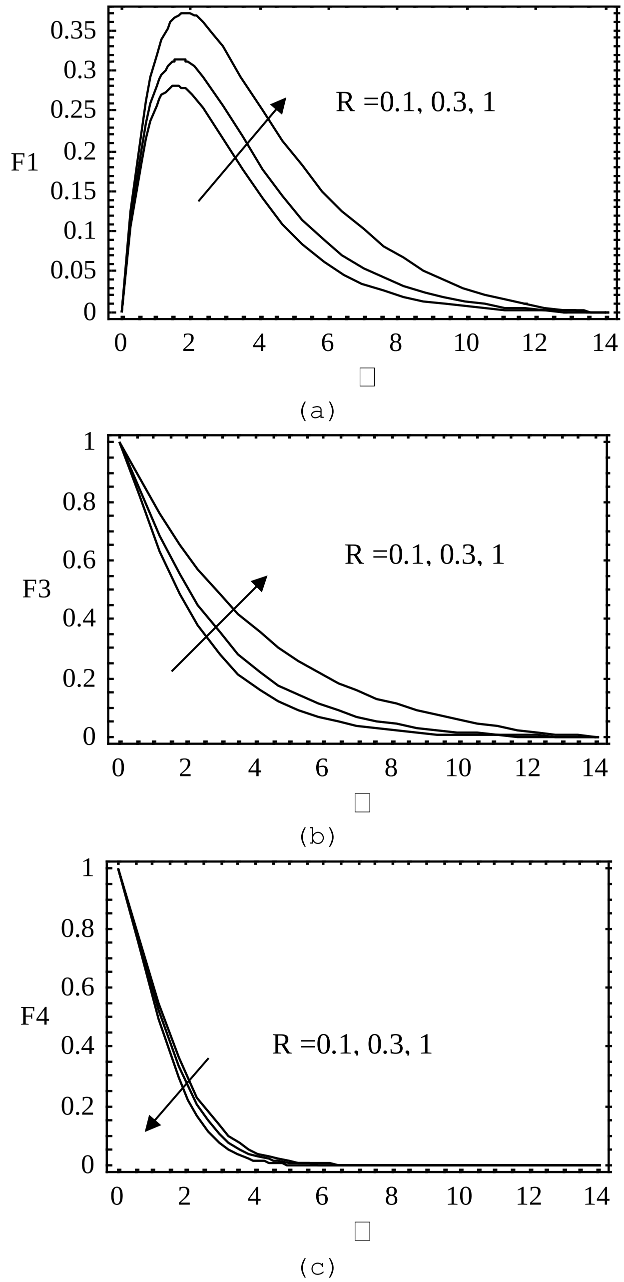
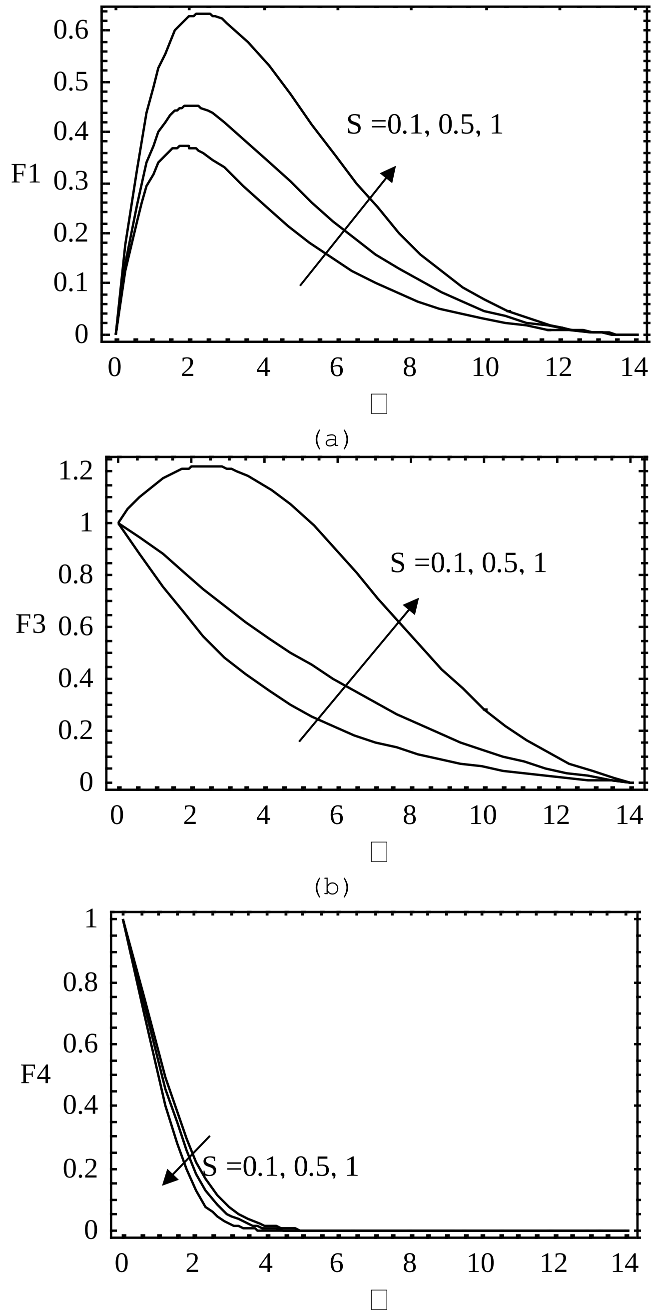


Figure 4 Velocity and temperature profiles for different R, Sc=1 Pr=0.71, Gc=0.1, K =5, S =0.1 and Gr =1



(c)

Figure 5 Velocity and temperature profiles for different S, Sc=1 Pr=0.71, Gc=0.1, K =5, R =1 and Gr =1

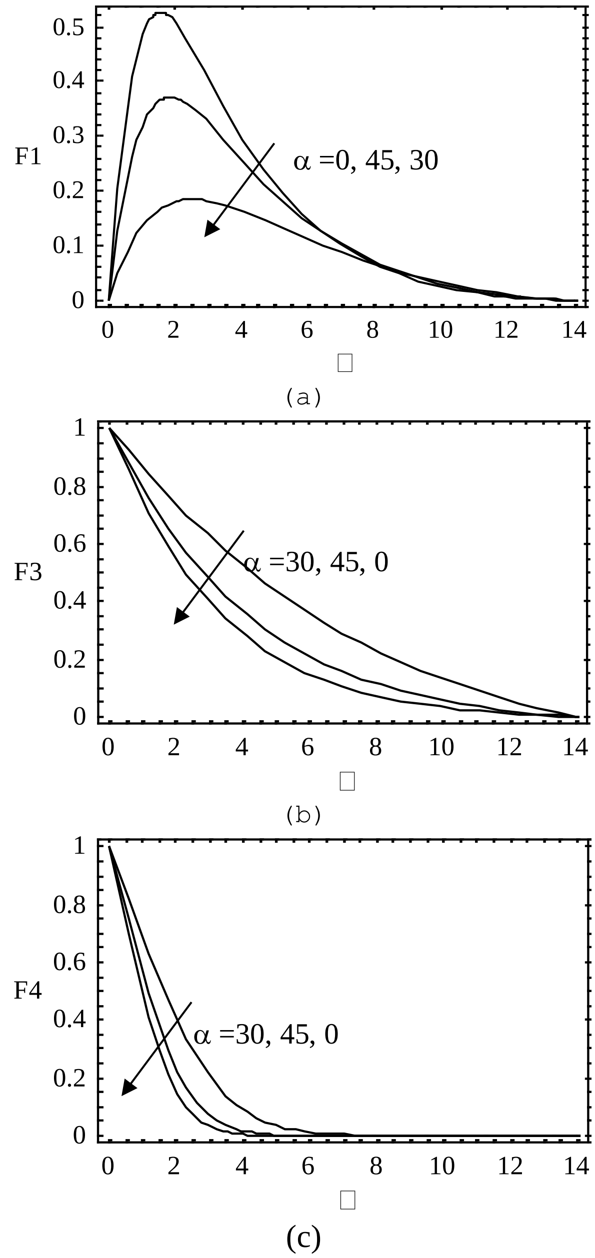
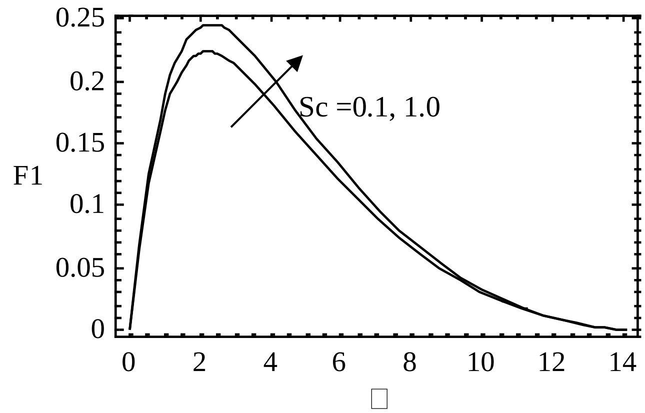
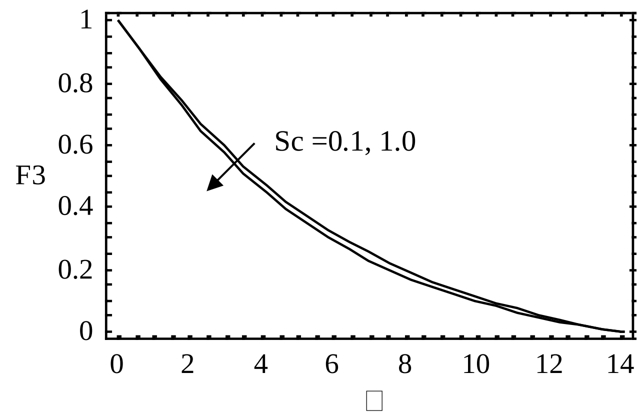


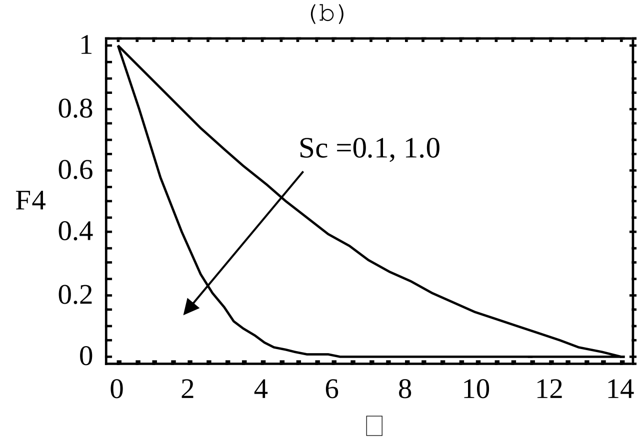
Figure 6 Velocity and temperature profiles for different α, Sc=1 Pr=0.71, Gc=0.1, K =5, S=0.1, R =1 and Gr =1.



(a)



(b)



(c)

Figure 7 Velocity and temperature profiles for different Sc, Pr=0.71, Gc=0.1, K =5, S=0.1, R =1 and Gr =1.

REFERENCES

1. Bluman GW and Kumei S. Symmetries and Differential Equations SpringerVerlag, 1989.
2. Chen CH. Heat and mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and concentration. Acta Mechanica. 2004;172; 219-235.
3. Megahed AA, Komy SR and Afify AA. Similarity analysis in magnetohydrodynamics: Hall effects on free convection flow and mass transfer past a semi-infinite vertical flat plate. Int. J. Non-linear Mech. 2003; 38; 513-520.
4. Molla MM, Taher MA, Chowdhury MMK and Hossain MA. Magnetohydrody-namic natural convection flow on a sphere in presence of heat generation. Non-linear Anal: Model. Con. 2005; 10 (4); 349-363.
5. Ibrahim FS, Mansour MA and Hamad MAA. Lie-group analysis of radiative and magnetic field effects on free convection and mass transfer flow past a semi-infinite vertical flat plate. Electronic J. Differential Equation. 2005; 39;1-17.
6. Kalpakides VK and Balassas KG. Symmetry groups and similarity solutions for a free convective boundary-layer problem. Int. J. Non-linear Mech. 2004; 39; 1659- 1670,.
7. Yurusoy M and Pakdemirli M. Symmetry reductions of unsteady threedimensional boundary layers of some non-Newtonian Fluids. Int. J. Engg. Sci. 1997; 35(8); 731-740,.
8. Yurusoy M., Pakdemirli M. Exact solutions of boundary layer equations of a special non Newtionian fluid over a stretching sheet. Mech. Res. Comm. 1999; 26 (2); 171-175.