Optimization of Multi-Objective Transportation Problems Using Membership Functions and Generalized Mean Operators

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Abstract: *This study addresses a multi-objective linear programming problem involving four conflicting objective functions-transportation cost, transportation time, fuel consumption, and carbon emissions across three sources and four destinations. The primary goal is to minimize all these objectives simultaneously to achieve an efficient and sustainable transportation plan, lo handle the inherent uncertainty and imprecision in decision-making, fuzzy membership functions are applied to convert the highest crisp objective values into lower values, enabling more flexible optimization. Subsequently, three distinct aggregation techniques Arithmetic Mean, Harmonic Mean, and Geometric Mean are employed to transform the multi objective problem into a single-objective formulation. This approach facilitates the use of standard optimization techniques for solution derivation. The study further performs a comparative analysis of the results obtained from these aggregation methods to identify the most effective strategy for balancing and optimizing the multiple objectives. Findings highlight the trade offs between different methods and provide insights into their suitability for practical transportation optimization problems, contributing valuable knowledge for decision makers in logistics and supply chain management focused on cost-efficiency and environmental sustainability.*

Keywords: **Multi-Objective Transportation Problem, Fuzzy Membership Function, Statistical Mean Approach.**

1. INTRODUCTION

Over the past few years, the multi-objective transportation problem (MOTP) has gained much interest owing to the development of optimization techniques and the growing size and complexity of logistics networks. As opposed to the traditional transportation problem, MOTP involves multiple objectives, e.g., minimizing cost and time, improving service quality, and lowering environmental impact, making it extremely viable in today's supply chain and logistics operations. Over time, researchers have come up with various techniques that blend traditional optimization with sophisticated methods such as metaheuristics, fuzzy systems, and evolutionary algorithms. For instance, Wang et al. [7] and Xie et al. [9] came up with better algorithms that made use of Pareto-optimality, enabling decision-makers to analyze multiple optimal trade-offs.

The development from single-purpose to multi-purpose models has been significantly motivated by the growing complexity of transport systems. Wang et al. [7] illustrated how particle swarm optimization (PSO) and genetic algorithms (GAs) greatly improve non-dominated solution generation of conflicting objectives like cost, time, and emissions. To boost computational efficiency in managing multiple objectives, Kumar and Panneerselvam [4] developed a hybrid approach integrating ant colony optimization (ACO) with linear programming. The performance of such strategies in managing complex and large-scale transport situations was further established by Xu et al. [8] and Bian et al. [2].

Fuzzy and probabilistic models were applied by scholars due to uncertainties in supply and demand. Agarwal and Aggarwal [1] proposed a fuzzy goal programming (FGP) approach to handle imprecise information in MOTPs. Zhu et al. [11] also dealt with uncertainties such as volatile fuel prices and traffic congestion through stochastic models, which allow for the creation of more robust transportation plans. Increasing emphasis on sustainability in supply chain management has also influenced MOTP studies. For example, Mousavi et al. [5] developed a green transport model that simultaneously reduces ecological and financial expenses, highlighting the compromise between environmental and economic needs.

To facilitate the assessment of multiple goals, researchers have used multi-criteria decision-making (MCDM) techniques such as the Analytic Hierarchy Process (AHP) and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). Chen et al. [3] described how MCDM methods might be used to balance stakeholder desires in MOTP uses and enhance decisional quality within both public and private transportation networks. Applications of MOTP within the real world also have appeared. For instance, Singh et al. [6] used a multi-objective approach to find optimum bus routes in Indian metropolises for operation cost and travel time. Zhou et al. [10] resolved the problem of freight delivery in urban areas by accommodating environmental objectives like CO2 minimization along with economic performance.

In the present research, we deal with the solution of a multi-objective transportation problem consisting of three supply centers and four destinations. The primary goals are reducing the cost of transportation, decreasing transit time, and decreasing the risk involved in the movement of goods. A balanced model of MOTP has been formulated in order to depict the methodology.

2. PRELIMINARIES

2.1 Multi-Objective Transportation Problem:
A Multi-Objective Transportation Problem (MOTP) is an extension of the classical transportation problem where multiple, often conflicting; objectives are optimized simultaneously. In this context, we are considering three objectives.

Objectives:

Let $o_{ij}^1, o_{ij}^2, o_{ij}^3, \cdots o_{ij}^r$ represent the cost coefficients associated with the r objectives for transporting from source i to destination j.

The multi-objective transportation problem can be stated as:

$$Min\ or\ Max\ Z_1 = \sum_{i=1}^{p}\sum_{j=1}^{q} o_{ij}^1 q_{ij}$$

$$Min\ or\ Max\ Z_2 = \sum_{i=1}^{p}\sum_{j=1}^{q} o_{ij}^2 q_{ij}$$

$$Min\ or\ Max\ Z_3 = \sum_{i=1}^{p}\sum_{j=1}^{q} o_{ij}^3 q_{ij}$$

$$\vdots \qquad \vdots$$

$$Min\ or\ Max\ Z_r = \sum_{i=1}^{p}\sum_{j=1}^{q} o_{ij}^r q_{ij}$$

Where Z_1, Z_2, Z_3 …… Z_r are the three objectives to be minimized or maximized.

Decision Variables:

Let q_{ij} represent the amount of goods transported from source i to destination j, where:

$i = 1, 2, \cdots, p$ (number of sources)

$j = 1, 2, \cdots, q$ (number of destinations)

Constraints:

1. ***Supply Constraints***:

$$\sum_{j=1}^{q} q_{ij} \leq s_i \qquad \forall\, i = 1, 2, \cdots, p$$

where s_i is the supply available at source i.

2. ***Demand Constraints***:

$$\sum_{i=1}^{p} q_{ij} \geq d_j \qquad \forall\, j = 1, 2, \cdots, q$$

where d_j is the supply available at source j.

3. ***Non-Negativity Constraints***:

$$q_{ij} \geq 0 \qquad \forall\, i = 1, 2, \cdots, p;\ j = 1, 2, \cdots, q$$

2.2 Fuzzy Membership function: Fuzzy membership function to convert the objectives into membership values to minimize a set of objectives. The membership function is defined as

$$M_r(x_{ij}^r) = \begin{cases} 1, x_{ij}^r \leq L_r \\ \frac{U_r - x_{ij}^r}{U_r - L_r}, L_r \leq x_{ij}^r \leq U_r \\ 0, x_{ij}^r \geq U_r \end{cases} \quad \ldots\ldots\ldots(1)$$

Where L_r is the lowest crisp value of x_{ij}^r and U_r is the highest crisp value of x_{ij}^r

2.3 Working Procedure:

Working Processor to solve MOTP as follows:

Step 1: Firstly, we checked the our MOTP is balanced. i.e. $\sum_{j=1}^{q} d_j = \sum_{i=1}^{p} s_i$

We move on step 3.

Step 2: If our MOTP is not balanced. i.e. $\sum_{j=1}^{q} d_j \neq \sum_{i=1}^{p} s_i$

according condition, we are going to add dummy row or column to convert out problem in balanced problem.

Step 3: Apply following Statistical Mean approaches to convert multi-objective into a single objective by using the following formulas:

- Arithmetic Mean.
- Harmonic Mean.
- Geometric Mean

Step 4: Apply the Vogel Approximation Method and MODI Methods to obtained the feasible solution and optimal solution respectively for obtained new objective functions.

Step 5: Find the optimal solution of all the objective functions of MOTP by using allocations which obtained from Step 4.

3. Numerical Problem

An agro-processing company in Madhya Pradesh needs to transport agricultural raw materials from various farms to processing plants while optimizing multiple objectives. The company sources wheat from four different farms and distributes it to three processing plants. The key objectives considered in transportation planning are:

1. Minimization of Transportation Cost - Reducing the cost of shipping raw materials.
2. Minimization of Transportation Time - Ensuring timely delivery to maintain quality.

3. Minimization of Transportation Time- Ensuring minimize the fuel consumption over all distribution
4. Minimization of Environmental Impact - Lowering carbon emissions from transportation.

Table 1: Sources and Destinations

Sources			Destination		
Farm	**City**	**Supply**	**Plant**	**City**	**Demand**
A	Mumbai	100	X	Ahmedabad	110
B	Delhi	120	Y	Hyderabad	130
C	Kolkata	80	Z	Benga	150
D	Chennai	90			

Table 2: Objective Matrix of Transportation Problem

Source→ Destination	**Cost (Rs Per Ton)** (Z_1)	**Time (Hours)** (Z_2)	**Fuel Consumption (Per Ton)**	**Carbon Emissions (Kg CO_2 Per ton)** (Z_3)
Farm A → Plant X	24	12	8	20
Farm A → Plant Y	30	14	10	30
Farm A → Plant Z	28	15	9	25
Farm B → Plant X	22	10	7	22
Farm B → Plant Y	26	13	9	28
Farm B → Plant Z	27	14	10	35
Farm C → Plant X	30	16	11	40
Farm C → Plant Y	25	15	9	25
Farm C → Plant Z	33	17	11	20
Farm D → Plant X	32	18	12	30
Farm D → Plant Y	20	10	8	45
Farm D → Plant Z	18	11	7	28

Now apply definition of membership function [equation 1] to minimize the objective values then obtained the following tables as follows

Table 3: Objective Matrix of implementation of membership function

Source→ Destination	**Cost (Rs Per Ton)** (Z_1)	**Time (Hours)** (Z_2)	**Fuel Consumption (Per Ton)** (Z_3)	**Carbon Emissions (Kg CO_2 Per ton)** (Z_4)
Farm A → Plant X	0.60	0.75	0.80	0.73
Farm A → Plant Y	0.20	0.50	0.40	0.27
Farm A → Plant Z	0.33	0.38	0.60	0.55
Farm B → Plant X	0.73	1.00	1.00	0.91
Farm B → Plant Y	0.47	0.63	0.60	0.45
Farm B → Plant Z	0.40	0.50	0.40	0.36
Farm C → Plant X	0.20	0.25	0.20	0.18
Farm C → Plant Y	0.53	0.38	0.60	0.55
Farm C → Plant Z	0.00	0.13	0.20	0.00
Farm D → Plant X	0.07	0.00	0.00	0.09

Farm D → Plant Y	0.87	1.00	0.80	0.82
Farm D → Plant Z	1.00	0.88	1.00	1.00

CASE I: Now apply the formula of arithmetic mean to change the all the objective function the single objective function, the obtained the table as follows

Table 4: Transportation Table after the implementation of arithmetic mean formula

	Ahmedabad	**Hyderabad**	**Bengaluru**	**Capacities**
Mumbai	0.72	0.34	0.46	**100**
Delhi	0.91	0.54	0.42	**120**
Kolkata	0.21	0.51	0.08	**80**
Chennai	0.04	0.87	0.97	**90**
Required	**110**	**130**	**150**	**390**

Here, apply the Vogel Approximation Method and MODI method to get the feasible and optimal solution respectively then we obtained the allocation as follows
$x_{12} = 100$ $x_{22} = 30$ $x_{23} = 90$ $x_{31} = 20$ $x_{33} = 60$ $x_{41} = 90$

and optimal solution of above problem as follows

$$Z_{HM}^{*} = 100 * 0.34 + 30 * 0.54 + 90 * 0.42 + 20 * 0.21 + 60 * 0.08 + 90 * 0.04 = 100.4$$

Now obtained optimal solution of all the define objectives of MOTP, which is mentioned below

Table 5: Optimal Solution according to arithmetic mean

Objective	Name of Objective	Optimal Solution
Z_1	Transportation Cost	Rs 11670
Z_2	Transportation Time	92 hours
Z_3	Fuel Consumption	4130 Letter
Z_4	Carbon Emission	9980 kg CO_2

CASE II: Now apply the formula of harmonic mean to change the all the objective function the single objective function, the obtained the table as follows

Table 6: Transportation Table after the implementation of Harmonic mean formula

	Ahmedabad	**Hyderabad**	**Bengaluru**	**Capacities**
Mumbai	0.71	0.30	0.44	**100**
Delhi	0.90	0.53	0.41	**120**
Kolkata	0.21	0.50	0.15	**80**
Chennai	0.08	0.86	0.97	**90**
Required	**110**	**130**	**150**	**390**

Here, apply the Vogel Approximation Method and MODI method to get the feasible and optimal solution respectively then we obtained the allocation as follows
$x_{12} = 100$ $x_{22} = 30$ $x_{23} = 90$ $x_{31} = 20$ $x_{33} = 60$ $x_{41} = 90$

and optimal solution of above problem as follows

$$Z^*_{HM} = 100 * 0.30 + 30 * 0.53 + 90 * 0.41 + 20 * 0.21 + 60 * 0.15 + 90 * 0.08 = 103.3$$

Now obtained optimal solution of all the define objectives of MOTP, which is mentioned below

Table 7: Optimal Solution according to Harmonic mean

Objective	Name of Objective	Optimal Solution
Z_1	Transportation Cost	Rs 11670
Z_2	Transportation Time	92 hours
Z_3	Fuel Consumption	4130 Letter
Z_4	Carbon Emission	9980 kg CO_2

CASE III: Now apply the formula of Geometric mean to change the all the objective function the single objective function, the obtained the table as follows

Table 8: Transportation Table after the implementation of Geometric mean formula

	Ahmedabad	**Hyderabad**	**Bengaluru**	**Capacities**
Mumbai	0.72	0.32	0.45	**100**
Delhi	0.90	0.53	0.41	**120**
Kolkata	0.21	0.51	0.16	**80**
Chennai	0.08	0.87	0.97	**90**
Required	**110**	**130**	**150**	**390**

Here, apply the Vogel Approximation Method and MODI method to get the feasible and optimal solution respectively then we obtained the allocation as follows
$x_{12} = 100$ $x_{22} = 30$ $x_{23} = 90$ $x_{31} = 20$ $x_{33} = 60$ $x_{41} = 90$

and optimal solution of above problem as follows

$$Z^*_{GM} = 100 * 0.32 + 30 * 0.53 + 90 * 0.41 + 20 * 0.21 + 60 * 0.16 + 90 * 0.08 = 106.4$$

Now obtained optimal solution of all the define objectives of MOTP, which is mentioned below

Table 9: Optimal Solution according to Geometric mean

Objective	Name of Objective	Optimal Solution
Z_1	Transportation Cost	Rs 11670
Z_2	Transportation Time	92 hours
Z_3	Fuel Consumption	4130 Letter

Z_4	Carbon Emission	9980 kg CO_2

Here, compiles all the optimal solution of objectives of MOTP and obtained objective according to applicable method and in tabular form as follows

Table 10: Optimal Solution of obtained Objectives

Objective	Name of Methods	Optimal Solution
Z_{AM}^{*}	Arithmetic Mean	100.4
Z_{HM}^{*}	Harmonic Mean	103.3
Z_{GM}^{*}	Geometric Mean	106.4

Table 10 presents the optimal solutions obtained using different aggregation methods-Arithmetic Mean Harmonic Mean, and Geometric Mean in the context of a multi-objective transportation problem. These methods help in consolidating multiple objectives into a single value for optimization. The resulting optimal solutions are 100.4 for the Arithmetic Mean, 103.3 for the Harmonic Mean, and 106.4 for the Geometric Mean. This table highlights how the choice of aggregation method can influence the final optimization outcome.

Table 11: Optimal Solution of all the Objectives

Objective	Name of Objective	Arithmetic Mean	Harmonic Mean	Geometric Mean
Z_1	Transportation Cost	Rs 11670	Rs 11670	Rs 11670
Z_2	Transportation Time	92 hours	92 hours	92 hours
Z_3	Fuel Consumption	4130 Letter	4130 Letter	4130 Letter
Z_4	Carbon Emission	9980 kg CO_2	9980 kg CO_2	9980 kg CO_2

Table 11 outlines the optimal values for individual transportation objectives-Cost, Time, Fuel Consumption, and Carbon Emission under each of the three mean methods (Arithmetic, Harmonic, and Geometric) Interestingly, all methods yield the same results a transportation cost of Rs 11,670, time of 92 hours, fuel consumption of 4130 liters, and carbon emission of 9980 kg CO, This consistency suggests that the methods converge effectively despite using different means.

4. RESULT AND DISCUSSION

The multi-objective transportation problem was addressed using three different aggregation methods-Arithmetic Mean, Harmonic Mean, and Geometric Mean. The optimal solutions from each method varied slightly, indicating sensitivity to the choice of mean. However, the core transportation objectives (cost, time, fuel consumption, and carbon emissions) remained consistent across methods. This consistency implies that while aggregation techniques may influence the scalar optimization value, they do not alter the actual operational outcomes, confirming the robustness and reliability of the optimized transportation plan

5. CONCLUSION

Multi-Objective Transportation Problems help industries make informed and balanced decisions by optimizing multiple criteria such as cost, time, fuel usage, and emissions. By applying MOTP techniques, industries can enhance supply chain efficiency, reduce operational costs, comply with environmental standards, and ensure timely deliveries-leading to sustainable and data-driven logistics strategies. This study presents an optimized solution to the Multi-Objective Transportation Problem (MOTP) using membership functions integrated with mean-based approaches—Arithmetic Mean, Harmonic Mean, and Geometric Mean. The results derived in Tables 10 and 11 demonstrate that all three methods yield consistent values for transportation cost (₹11,670), time (92 hours), fuel consumption (4,130 liters), and carbon emission (9,980 kg CO_2). However, the optimal values computed via membership functions show slight variations: Arithmetic Mean (100.4), Harmonic Mean (103.3), and Geometric Mean (106.4), highlighting their influence on decision-making preferences. These variations offer flexibility in optimization, enabling the decision-maker to select a solution aligned with specific operational goals, such as cost-efficiency or environmental sustainability. The framework ensures balanced trade-offs among multiple conflicting objectives, proving useful for real-world logistics and transportation systems.

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