

# The Notion of new mappings in Minimal Structure

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**Abstract**— This paper aims forming at the some mappings like  $m_X$ -feebly regular open with its complement mapping. These concepts are defined at the  $m_X$ -feebly regular continuous function and also discussed at the some related theorems in it.

**Keywords**—  $m_X$ -feebly open,  $m_X$ -feebly closed,  $m_X$ -feebly interior,  $m_X$ -feebly closure,  $m_X$ -feebly clopen,  $m_X$ -feebly regular open and  $m_X$ -feebly regular closed.

## I INTRODUCTION

General topology is the main role of mathematical field. In 1963, Levine introduced at the concepts of semi open set and semi-continuous. The semi open sets, preopen sets,  $\alpha$ -open sets,  $\beta$ -open sets, b-open sets and  $\delta$ -open sets play an important role in the research of generalization of continuity in topological spaces. By using these sets several authors introduced at the various types of Non-continuous functions. Further the analogy in their definitions and properties suggests the need of formulating in the setting of functions. In 1982 Tong., J investigated at the separation axioms and decomposition of continuity. In 1982, S.N Maheswari and P.C. Jain defined and studied at the concepts of feebly open and feebly closed sets in topological spaces. In 2000, the concepts of minimal structure (briefly  $m_X$ -structure) was introduced by V. Popa and T. Noiri. They introduced at the notions of  $m_X$ -open sets and  $m_X$ -closed sets and characterize of those sets using  $m_X$ -closure and  $m_X$  operators, respectively and also obtained the definitions and characterizations of some mappings by using at the concept of minimal structure.

## II

### PRELIMINARIES

**Definition 2.1:** Let  $(X, \tau)$  be a topological space. A subset  $A$  of  $X$  is said to be

- 1)  $\alpha$ -open [9] if  $A \subset \text{int}(\text{cl}(\text{int}(A)))$
- 2) Semi-open [6] if  $A \subset \text{cl}(\text{int}(A))$
- 3) Preopen [9] if  $A \subset \text{int}(\text{cl}(A))$
- 4) b-open [2] if  $A \subset \text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A))$
- 5)  $\beta$ -open [1] or semi-preopen if  $A \subset \text{cl}(\text{int}(\text{cl}(A)))$
- 6) Feebly open [7] if  $A \subset s \text{cl}(\text{int}(A))$
- 7) Feebly closed [7] if  $\text{int}(\text{cl}(A)) \subset A$

The family of all  $\alpha$ -open (resp., semi-open, preopen, b-open,  $\beta$ -open, feebly open, feebly closed) sets in  $(X, \tau)$  is denoted by  $\alpha(X)$  (resp.,  $SO(X)$ ,  $PO(X)$ ,  $BO(X)$ ,  $\beta(X)$ ,  $FO(X)$ ).

**Definition 2.2 [11, 12]:** A subfamily  $m_X$  of the power set  $P(X)$  of a non-empty set  $X$  is called a minimal structure (briefly m-structure) on  $X$  if  $\emptyset \in m_X$  and  $X \in m_X$ . By  $(X, m_X)$  we denote a non-empty set  $X$  with a minimal structure  $m_X$  on  $X$  and call it an m-space. Each member of  $m_X$  is said to be  $m_X$ -open and the complement of an  $m_X$ -open is said to be  $m_X$ -closed.

**Remark 2.3:** Let  $(X, \tau)$  be a topological space. Then the families  $\tau$ ,  $SO(X)$ ,  $PO(X)$ ,  $BO(X)$  and  $\beta(X)$  are all  $m$ -structure on  $X$ .

**Definition 2.4:** Let  $(X, m_X)$  be a  $m_X$ -space. For a subset  $A$  of  $m_X$ -closure of  $A$  and  $m_X$ -interior of  $A$  are defined in [8] as follows:

- (i)  $m_X\text{-cl}(A) = \bigcap \{F : A \subset F, X - F \in m_X\}$
- (ii)  $m_X\text{-int}(A) = \bigcup \{U : U \subset A, U \in m_X\}$

**Remark 2.5:** Let  $(X, \tau)$  be a topological space and let  $A$  be a subset of  $X$ . If  $m_X = \tau$  (resp.,  $so(X), PO(X), \alpha(X), BO(X)$  and  $\beta(X)$ ) then we have

- (a)  $m_X\text{-cl}(A) = \text{cl}(A)$  (resp  $S.\text{cl}(A), P.\text{cl}(A), \alpha.\text{cl}(A), b.\text{cl}(A)$  and  $\beta.\text{cl}(A)$ ).
- (b)  $m_X\text{-int}(A) = \text{int}(A)$  (resp  $S.\text{int}(A), P.\text{int}(A), \alpha.\text{int}(A), b.\text{int}(A)$  and  $\beta.\text{int}(A)$ ).

**Remark 2.6:** Let  $A$  be a subset of  $(X, m_X)$

- (a) The union of all  $m_X$ -semi open sets of  $X$  contained in  $A$  is called the  $m_X$ -semi-interior of  $A$ .
- (b) The intersection of all  $m_X$ -semi closed sets of  $X$  containing  $A$  is called the  $m_X$ -semi closure of  $A$ .
- (c)  $A$  is called  $m_X$ -semi preopen if  $A \subset m_X\text{-cl}(m_X\text{-int}(m_X\text{-cl}(A)))$ . Its complement is  $m_X$ -semi-preclosed.

**Definition 2.7 [7]:** A subset  $A$  of  $(X, m_X)$  is said to be  $m_X$ -regular open (briefly  $m_X$ -RO) if  $A = m_X\text{-int}(A)$  and  $m_X$ -regular closed if  $A = m_X\text{-cl}(A)$ .

**Definition 2.8 [12]:** Let  $f : (X, m_X) \rightarrow (Y, m_Y)$  be a function between a space  $(X, m_X)$  with minimal structure  $m_X$  and a topological space  $(Y, m_Y)$ . Then  $f$  said to be  $m$ -continuous if for each  $x$  and each open set  $V$  containing  $f(x)$ , there exists an  $m$ -open set  $U$  containing  $x$  such that  $f(U) \subseteq V$ .

**Definition 2.9 [5]:** Let  $(X, m_X)$  be a topological space. Any subset  $A$  of  $X$  is called feebly clopen if it is both feebly open and feebly closed.

**Definition 2.10:** A map  $f : X \rightarrow Y$  is said to be

1. Feebly closed (resp., feebly open) [10] if the image of each closed set (resp., open set) in  $X$  is feebly closed (resp., feebly open) set in  $Y$
2. Feebly continuous [10] if  $f^{-1}(V)$  is feebly open in  $X$  for each set  $V$  of  $Y$ .
3. Feebly clopen [5] if the image of every open and closed set in  $X$  is both feebly open and closed in  $Y$ .

**Definition 2.11[3]:** A subset  $(E, m_X)$  of a  $m_X$ -space  $(X, m_X)$  is said to be  $m_X$ -feebly open if  $E \subset m_X\text{-s cl}(m_X\text{-int}(E))$ .

**Definition 2.12[3]:** A subset  $(E, m_X)$  of a  $m_X$ -space  $(X, m_X)$  is said to be  $m_X$ -feebly closed if  $m_X\text{-int}(m_X\text{-cl}(E)) \subset E$ .

**Definition 2.13[3]:** The  $m_X$ -feebly closure of  $(E, m_X)$  is the intersection of all  $m_X$ -feebly closed set containing  $(E, m_X)$  and is denoted by  $m_X\text{-f.cl}(E)$ .

**Definition 2.14[3]:** The  $m_X$ -feebly interior of  $(E, m_X)$  is the union of all  $m_X$ -feebly open sets contained in  $(E, m_X)$  and is denoted by  $m_X\text{-f.int}(E)$ .

**Definition 2.15[13]:** A subset  $A$  of  $(X, m_X)$  is said to be  $m_X$ -Feebly regular open (briefly  $m_X\text{-f.reg.open}$ ) if  $A = m_X\text{-f.int}(m_X\text{-f.cl}(A))$ .

**Definition2.16[13]:**A subset A of  $(X, m_X)$  is said to be  $m_X$  – **Feebly regular closed** if  $A = m_X -f.cl (m_X -f.int (A))$  (briefly  $m_X -f.reg.closed$ ).

**Definition2.17[13]:**A subset A of  $(X, m_X)$  is said to be  $m_X$  – **Feebly regular clopen** if  $A = m_X -f.int (m_X -f.cl (m_X -f.int (A)))$ . On the other hand, if A is  $m_X -f.reg.open$  and  $m_X -f.reg.closed$ .

**Definition2.18[13]:**Let A be a subset of  $(X, m_X)$ . The  $m_X$  – **Feebly regular closure** of A (briefly  $m_X -f.reg.cl(A)$ ) is the intersection of all  $m_X$  – Feebly regular closed sets containing A and the  $m_X$  –**Feebly regular interior** of A (briefly  $m_X -f.reg.int(A)$ ) is the union of all  $m_X$  – Feebly regular open sets contained in A. The complement of  $m_X$  – Feebly regular open set is  $m_X$  – Feebly regular closed.

### III $m_X$ – Feebly regular open mappings with its complement

In this section we introduce  $m_X$  – Feebly regular open and  $m_X$  – Feebly regular closed mappings and some of its properties are discussed.

**Definition 3.1:** A function  $f : (X, m_X) \rightarrow (Y, m_Y)$  is called the  $m_X$  – Feebly regular closed if the image of each  $m_X$  – Feebly regular closed in  $(X, m_X)$  is a  $m_X$  – Feebly regular closed in  $(Y, m_Y)$ .

**Definition 3.2:** A function  $f : (X, m_X) \rightarrow (Y, m_Y)$  is called  $m_X$  – Feebly regular open if the image of each  $m_X$  – open set in  $(X, m_X)$  is  $m_X$  – Feebly regular open set in  $(Y, m_Y)$ .

**Theorem 3.3:** Every  $m_X$  – open mapping is  $m_X$  – Feebly regular open mapping.

**Proof:** Let  $f : (X, m_X) \rightarrow (Y, m_Y)$  be a  $m_X$  – open mapping. Now we have to prove that f is  $m_X$  – Feebly regular open. Let H be any  $m_X$  – open subset of  $(X, m_X)$  . Since f is  $m_X$  – open mapping,  $f(H)$  is  $m_X$  – open in  $(Y, m_Y)$  ,  $f(H)$  is  $m_X$  – Feebly regular open. Hence f is  $m_X$  – feebly open mapping.

**Theorem 3.4:** Every  $m_X$  – closed mapping is  $m_X$  – Feebly regular closed mapping.

**Proof:** Let  $f : (X, m_X) \rightarrow (Y, m_Y)$  be a  $m_X$  – closed mapping. Now we have to prove that f is  $m_X$  – Feebly regular closed mapping. Let H be any  $m_X$  – closed subset of  $(X, m_X)$  . Since f is  $m_X$  – closed mapping,  $f(H)$  is  $m_X$  – closed in  $(Y, m_Y)$  .  $f(H)$  is  $m_X$  – Feebly regular closed. Hence f is  $m_X$  – Feebly closed mapping.

**Theorem 3.5:** Let  $f : (X, m_X) \rightarrow (Y, m_Y)$  be a  $m_X$  – Feebly regular closed mapping then the image of every  $m_X$  – closed subset of  $(X, m_X)$  is  $m_X$  – Feebly regular closed in  $(Y, m_Y)$  .

**Proof:** Let H be any  $m_X$  – closed subset of  $(X, m_X)$  and  $f(H)$  is  $m_X$  – Feebly regular closed in  $(Y, m_Y)$  then  $f(H)$  is  $m_X$  – Feebly regular closed in  $(Y, m_Y)$  .

**Theorem 3.6:** A mapping  $f : (X, m_X) \rightarrow (Y, m_Y)$  is  $m_X$  – Feebly regular open if  $f(m_X -F.reg.int(H)) \subseteq m_X -F.reg.int f(H)$  for every  $H \subseteq (X, m_X)$  .

**Proof:** Let H be any  $m_X$  – open set in  $(X, m_X)$  . So that  $m_X -F.reg.int (H) = H$ , then  $f(m_X -F.reg.int( H)) \subseteq m_X -F.reg.int f(H)$ . Therefore  $f(H) \subseteq m_X -F.reg.int f(H)$ . But  $m_X -F.reg.int (f(H)) \subseteq f(H)$  always. Hence  $m_X -F.reg.int (f(H)) = f(H)$ . Therefore  $f(H)$  is  $m_X$  – open in  $(X, m_X)$  , then f is  $m_X$  – open. By theorem 3.3, f is  $m_X$  – Feebly regular open.

**Theorem 3.7:** A mapping  $f : (X, m_X) \rightarrow (Y, m_Y)$  is  $m_X$  – Feebly regular closed if  $m_X$  –F.reg.cl ( $f(H)$ )  $\subset f(m_X$  –F.reg.cl (H)) for every  $H \subset (X, m_X)$ .

**Proof:** Let H be any  $m_X$  – closed set in  $(X, m_X)$ . So that  $m_X$  –F.reg.cl(H) = H. By hypothesis  $m_X$  – F.reg.cl  $f(H) \subset f(m_X$  –F.reg.cl (H))= $f(H)$ . Therefore,  $m_X$  –F.reg.cl ( $f(H)$ )  $\subset f(H)$ . But  $f(H) \subset m_X$  – F.reg.cl  $f(H)$  always. Hence  $m_X$  –F.reg.cl  $f(H)=f(H)$ , thus  $f(H)$  is  $m_X$  – closed, then  $f$  is  $m_X$  – closed map. By theorem 3.4,  $f$  is  $m_X$  – Feebly regular closed.

**Theorem 3.8:** Let  $f : (X, m_X) \rightarrow (Y, m_Y)$  and  $g : (Y, m_Y) \rightarrow (Z, m_Z)$  be a mappings then  $g \circ f : (X, m_X) \rightarrow (Z, m_Z)$  is  $m_X$  – Feebly regular open if (i)  $f$  and  $g$  be the  $m_X$  –open mappings (ii)  $f$  is  $m_X$  – open and  $g$  is  $m_X$  – Feebly regular open mappings.

**Proof:** (i) Let H be any  $m_X$  –open subset of  $(X, m_X)$ . Now we have to prove that  $(g \circ f) (H)$  is  $m_X$  – Feebly regular open in  $(Z, m_Z)$ . Since  $f$  is  $m_X$  –open, then  $f(H)$  is  $m_X$  –open in  $(Y, m_Y)$ . Also we have  $g$  is  $m_X$  – open, then  $g(f(H))$  is  $m_X$  –open in  $(Z, m_Z)$ . Therefore  $(g \circ f) (H)$  is  $m_X$  – Feebly regular open in  $(Z, m_Z)$ . Thus  $g \circ f$  is  $m_X$  – Feebly regular open mapping.

(ii) By same method in part (i).

**Remark 3.9:** (i) Let  $f : (X, m_X) \rightarrow (Y, m_Y)$  and  $g : (Y, m_Y) \rightarrow (Z, m_Z)$  be the  $m_X$  –closed mappings then  $g \circ f : (X, m_X) \rightarrow (Z, m_Z)$  is  $m_X$  – Feebly regular closed. (ii) Let  $f : (X, m_X) \rightarrow (Y, m_Y)$  be  $m_X$  –closed and  $g : (Y, m_Y) \rightarrow (Z, m_Z)$  be  $m_X$  – Feebly regular closed, then  $g \circ f : (X, m_X) \rightarrow (Z, m_Z)$  is  $m_X$  – Feebly regular closed.

#### 4. $m_X$ – Feebly regular continuous functions

**Definition 4.1:** A  $m_X$  –mapping  $f : (X, m_X) \rightarrow (Y, m_Y)$  is said to be  $m_X$  – Feebly regular continuous if the soft inverse image by  $f$  of each  $m_X$  – open set H of  $(Y, m_Y)$  is  $m_X$  – Feebly regular open in  $(X, m_X)$ .

**Remark 4.2:** (i) Let  $(X, m_X)$  be a  $m_X$  – topological space, A and  $B \subset (X, m_X)$  if  $A \subset B$ , then  $f(m_X$  –F.reg.cl (A))  $\subset f(m_X$  –F.reg.cl (B)). (ii) Let  $(X, m_X)$  be a minimal topological space if A is  $m_X$  – Feebly regular open if and if only  $A^c$  is  $m_X$  – Feebly regular closed. From our definition of  $m_X$  – Feebly regular open and  $m_X$  – Feebly regular closed sets, we obtain them.

**Theorem 4.3:** If  $f : (X, m_X) \rightarrow (Y, m_Y)$  is  $m_X$  – Feebly regular continuous if and if only the inverse image of every  $m_X$  –closed subset of  $(Y, m_Y)$  is  $m_X$  – Feebly regular closed in  $(X, m_X)$ .

**Proof:** We have  $f$  is  $m_X$  – Feebly regular continuous. Let H is  $m_X$  –closed in  $(Y, m_Y)$ ,  $H^c$  is  $m_X$  –open in minimal structure,  $f^1(H)^c = (f^1(H))^c$  is  $m_X$  – Feebly regular open in  $(X, m_X)$ , then by remark 4.2(ii)  $f^1(H)$  is  $m_X$  – Feebly regular closed. H is a  $m_X$  –open set in  $(Y, m_Y)$ ,  $H^c$  is  $m_X$  –closed, then by hypothesis  $f^1(H)^c$  is  $m_X$  – Feebly regular closed in  $(X, m_X)$ , then by remark 4.5(ii)  $f^1(H)$  is  $m_X$  – Feebly regular open set in  $(Y, m_Y)$ . Thus  $f$  is  $m_X$  – Feebly regular continuous.

**Theorem 4.4:** Every  $m_X$  –continuous mapping is  $m_X$  – Feebly regular continuous mapping.

**Proof:** Let  $f : (X, m_X) \rightarrow (Y, m_Y)$  is  $m_X$  – continuous mapping. Now we have to prove that  $f$  is  $m_X$  – Feebly regular continuous. Let H be any  $m_X$  – open subset of  $(Y, m_Y)$ . Since  $f$  is  $m_X$  –continuous then  $f^1(H)$  is  $m_X$  –open in  $(X, m_X)$ . Therefore  $f^1(H)$  is  $m_X$  – Feebly regular open. Hence  $f$  is  $m_X$  – Feebly regular continuous mapping.

**Theorem 4.5:** Let  $(X, m_X)$  be a  $m_X$ -topological space,  $A \subset (X, m_X)$ , then  $A$  is  $m_X$ -Feebly regular open if and if only  $f m_X$ -F.reg.int  $(A) = A$ .

**Proof:** We have  $A$  is  $m_X$ -Feebly regular open set in  $(X, m_X)$ . It is clear  $f(m_X$ -F.reg.int  $(A)) \subset A \rightarrow (1)$ . Since  $A$  is  $m_X$ -Feebly regular open set and  $f(m_X$ -F.reg.int  $(A))$  is largest  $m_X$ -Feebly regular open set. Then  $A \subset f(m_X$ -F.reg.int  $(A)) \rightarrow (2)$ . From (1) and (2) we obtain  $f(m_X$ -F.reg.int  $(A)) = A$ . Conversely let  $f(m_X$ -F.reg.int  $(A)) = A$ . Since  $f(m_X$ -F.reg.int  $(A))$  is  $m_X$ -Feebly regular open set, then  $A$  is  $m_X$ -Feebly regular open set.

**Corollary 4.6:** Let  $(X, m_X)$  be a  $m_X$ -topological space,  $A \subset (X, m_X)$  then  $A$  is  $m_X$ -Feebly regular closed if and if only  $f(m_X$ -F.reg.cl  $(A)) = A$ .

**Theorem 4.7:** If  $f : (X, m_X) \rightarrow (Y, m_Y)$  is  $m_X$ -Feebly regular continuous if and if only  $f(f(m_X$ -F.reg.cl  $(A))) \subset f(A^c)$  for every  $A \subset (X, m_X)$ .

**Proof:** We have  $f$  is  $m_X$ -Feebly regular continuous. Since  $f(A^c)$  is soft-closed in  $(Y, m_Y)$ , then by theorem 4.3,  $f^1(f(A^c))$  is  $m_X$ -Feebly regular closed in  $(X, m_X)$ . By corollary 4.6,  $f(m_X$ -F.reg.cl  $(f^1(f(A^c)))) = f^1(f(A^c)) \rightarrow (1)$ . Now  $f(A) \subset f(A^c) \Rightarrow A \subset f^1(f(A))$  then  $A \subset f^1(f(A^c))$ , thus by remark 4.2(i),  $f(m_X$ -F.reg.cl  $(A)) \subset f(m_X$ -F.reg.cl  $(f^1(f(A^c))))$  according to (1), we get  $f(m_X$ -F.reg.cl  $(A)) \subset f^1(f(A^c))$ , then  $f(f(m_X$ -F.reg.cl  $(A))) \subset (f(A^c))^c$ . Conversely, let  $f(f(m_X$ -F.reg.cl  $(A))) \subset (f(A))^c$  for every  $A \subset (X, m_X)$ . Let  $H$  is  $m_X$ -closed set in  $(Y, m_Y)$ . Then  $H^c = H$ , let  $f^1(H)$  be any  $m_X$ -subset of  $(X, m_X)$ , then by hypothesis  $f(m_X$ -F.reg.cl  $(f^1(A))) \subset f((f^1(H))^c) = H^c = H$ . Thus  $f(m_X$ -F.reg.cl  $(f^1(H))) \subset f^1(H)$  but  $f^1(H) \subset f(m_X$ -F.reg.cl  $(f^1(H)))$  always thus  $f^1(H) = f(m_X$ -F.reg.cl  $(f^1(H)))$ . Therefore by corollary 4.6,  $f^1(H)$  is  $m_X$ -Feebly regular closed in  $(X, m_X)$ , hence by theorem 4.6,  $f$  is  $m_X$ -Feebly regular continuous.

**Theorem 4.8:** If  $f : (X, m_X) \rightarrow (Y, m_Y)$  is  $m_X$ -Feebly regular continuous if and if only  $f(m_X$ -F.reg.cl  $(f^1(B))) \subset f^1(B^c)$  for every  $B \subset (Y, m_Y)$ .

**Proof:** We have  $f$  is  $m_X$ -Feebly regular continuous. Since  $B^c$  is  $m_X$ -closed in  $(Y, m_Y)$ . Then by theorem 4.3,  $f^1(B^c)$  is  $m_X$ -Feebly regular closed in  $(X, m_X)$  and by corollary 4.6,  $f(m_X$ -F.reg.cl  $(f^1(B^c))) = f^1(B^c) \rightarrow (1)$ . Now  $B \subset B^c \Rightarrow f^1(B) \subset f^1(B^c)$  then by remark 4.2(i),  $f(m_X$ -F.reg.cl  $(f^1(B))) \subset f(m_X$ -F.reg.cl  $(f^1(B^c)))$ , according to (1) we get,  $f(m_X$ -F.reg.cl  $(f^1(B))) \subset f^1(B^c)$ . Conversely, let  $f(m_X$ -F.reg.cl  $(f^1(B))) \subset f^1(B^c)$  for every  $B \subset (Y, m_Y)$ . Let  $H$  be any  $m_X$ -closed in  $(Y, m_Y)$ . Then  $H^c = H$  by hypothesis  $f(m_X$ -F.reg.cl  $(f^1(H))) \subset f^1(H^c) = f^1(H)$ . Thus  $f(m_X$ -F.reg.cl  $(f^1(H))) \subset f^1(H)$ , but  $f^1(H) \subset f(m_X$ -F.reg.cl  $(f^1(H)))$ , therefore  $f(m_X$ -F.reg.cl  $(f^1(H))) = f^1(H)$ . Then by corollary 4.6,  $f^1(H)$  is  $m_X$ -Feebly regular closed in  $(X, m_X)$ , hence by theorem 4.3,  $f$  is  $m_X$ -Feebly regular continuous.

**Theorem 4.9:** Let  $f : (X, m_X) \rightarrow (Y, m_Y)$  be a  $m_X$ -mapping if  $f(m_X$ -F.reg.cl  $(H)) \subset f(m_X$ -F.reg.cl  $(f(H)))$  for every  $H \subset (X, m_X)$  then  $f$  is  $m_X$ -Feebly regular continuous.

**Proof:** Let  $H$  be any  $m_X$ -closed set in  $(Y, m_Y)$ , then by remark 4.2(ii), let  $H$  is  $m_X$ -Feebly regular closed so that by corollary,  $f(m_X$ -F.reg.cl  $(H)) = H$ ,  $f^1(H)$  is a  $m_X$ -subset of  $(X, m_X)$  so that by hypothesis  $f(m_X$ -F.reg.cl  $(f^1(H))) \subset f(m_X$ -F.reg.cl  $(f(f^1(H)))) = f(m_X$ -F.reg.cl  $(H)) = H$ . Therefore  $f(m_X$ -F.reg.cl  $(f^1(H))) \subset f^1(H)$  always. Hence  $f(m_X$ -F.reg.cl  $(f^1(H))) = f^1(H)$  then by corollary 4.6,  $f^1(H)$  is  $m_X$ -Feebly regular closed in  $(X, m_X)$ . Therefore by theorem 4.3,  $f$  is  $m_X$ -Feebly regular continuous.

**Theorem 4.10:** Let  $f : (X, m_X) \rightarrow (Y, m_Y)$  be a  $m_X$ -mapping if  $f(m_X\text{-F.reg.cl}(f^{-1}(H))) \subset f^{-1}(f(m_X\text{-F.reg.cl}(H)))$  for every  $H \subset (Y, m_Y)$  then  $f$  is  $m_X$ -Feebly regular continuous.

**Proof:** Let  $H$  be any  $m_X$ -closed set in  $(X, m_X)$  then by result 3.5, we have  $H$  is a  $m_X$ -Feebly regular closed set and so by corollary 4.6,  $f(m_X\text{-F.reg.cl}(H)) = H$ . By hypothesis  $f(m_X\text{-F.reg.cl}(f^{-1}(H))) \subset f^{-1}(f(m_X\text{-F.reg.cl}(H))) = f^{-1}(H)$ . Therefore  $m_X\text{-F.reg.cl}(f^{-1}(H)) \subset f^{-1}(H)$ . But  $f^{-1}(H) \subset f(m_X\text{-F.reg.cl}(f^{-1}(H)))$  always. Hence  $f(m_X\text{-F.reg.cl}(f^{-1}(H))) = f^{-1}(H)$  then by corollary 4.6,  $f^{-1}(H)$  is  $m_X$ -Feebly regular closed in  $(X, m_X)$ . Therefore by theorem 4.3,  $f$  is  $m_X$ -Feebly regular continuous.

### III CONCLUSIONS

In further work, at these mappings based on these sets with its related points will be generalized and also extended.

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