Research Articles

A retailer's optimal decision for an integrated framework for managing advance payments and customer returns for price-sensitive demand

AMARAT H. RATHOD
Research Scholar, Gujarat Technological University, Ahmedabad, Gujarat, India
Lecturer, Department of Sciences and Humanities, Government Polytechnic for Girls, Ahmedabad, Gujarat, India

KUNAL TARUNKUMAR SHUKLA*
Assistant Professor, Department of Sciences and Humanities, Vishwakarma Government Engineering College, Ahmedabad, Gujarat, India
(Corresponding author)
DHARMESH K. KATARIYA
Assistant Professor, Department of Sciences and Humanities, Shantilal Shah Engineering College, Bhavnagar, Gujarat, India

Abstract

In today's competitive market, customers have become increasingly price-sensitive. The rise of e-commerce and social media has therefore increased customer returns from comparative price analysis. Customers returning a product to retailers can improve a business in several ways. Retailers play a crucial role in contact with customers, and they can promote the sale of goods through personal interaction. In the supply chain, securing payment for goods to suppliers is crucial. We develop a mathematical model to analyzes the effective and efficient effect of advance payments to suppliers, customers return, and inventory operations management from a retailer's perspective aiming to optimize revenue in a price-sensitive market. This study aims to determine the optimal selling price, order quantity, and retailer's profit, considering the impact of customer returns and advance payment. The global optimality of total average profit is verified using the Hessian matrix method. The result shows that when a full partial advance payment policy is implemented there is a significant increase in profit of 36% relative to partial advance payment with full equal installments and price elasticity significantly affects profit margin. This model is validated through numerical examples, and its managerial insights are derived from sensitivity analysis of gradually increasing solution performance. In the conclusion section, unique research insights, future research directions, and limitations and challenges of the study are shown.s

Keywords: Inventory, Customers return, Advance payment, Price-sensitive demand, Selling price, Cycle time.

1. Introduction

Customers are returning purchased items to retailers for many reasons and suppliers insist on prepayment to reduce non-payment risks, increase immediate cash flow and effectively navigate market volatility. This situation is very challenging in a competitive business environment. In the rivalry-driven business environment of today, stock viability and customer satisfaction are crucial. Handling customer returns, whether for product defects, wrong choices, sizing errors, or unmet expectations, fosters customer satisfaction, trust, and loyalty, ultimately enhancing brand image. The retailers who act as the last link in the supply chain play a crucial role in connecting brands with customers.

Our research focuses on the importance of supplier, customer, and retailer interconnectivity to optimize supply chain efficiency. Today, the strong linkages with manufacturers, retailers, and consumers, who are the main drivers of the supply chain, have attracted the attention of many researchers. Thus, in this article, we address three distinct areas of research: supplier payments, customer returns, and product deterioration.

The first relevant literature is research on customers return. Pasternack (1985) introduced optimal pricing and return policies for perishable commodities. Emmons *et al*. (1998) focused on the role of returns policies in pricing and inventory decisions making. He *et al*. (2006) also presented a return policy model of supply chain management for single-period products. Mostard and Teunter (2006) addressed a similar newsvendor problem with customer returns that arrived back before the end of the selling season they restricted attention to a fixed retail price and resold. Chen and Bell (2009) focused on how suppliers should set prices and manage inventory when a customer returns a product. Hu *et al*. (2012) focused on implementing return policies in two-echelon supply chain managements Ghoreishi *et al*. (2013a,2013b) presented a model to determine the optimal selling price, the optimal replenishment cycles, and the order quantity simultaneously under a price and time dependent demand function, also the customer returns are considered as a function of both price and demand. Also, developed an inventory model for joint optimal pricing and inventory control for deteriorating items under inflation and customer returns. Shah *et al*. (2015) also presented an economic order quantity model under trade credit and customer returns for price-sensitive quadratic demand.

The second relevant literature is research on ecumenic order quantity inventory model under advance payment policy. Zhang *et al*. (2014) discussed in detail for an economic order quantity under advance payment. Zia and Taleizadeh (2015) proposed a model demonstrating that sales can be optimally boosted through payment schemes tied to order quantities, incorporating multiple advance and deferred payments. Tavakoli and Taleizadeh (2017) also developed a buyer's inventory management system for deteriorating items with full advance payment, incorporating three distinct shortage scenarios. Nath *et al.* (2021) developed an inventory model for deteriorating imperfect items under advanced payment. Khan *et al.* (2022a, 2022b) adopted a practical scenario for inventory management with hybrid cash-advance payment for time-dependent demand, time-varying holding cost and non-instantaneous deterioration under backordering and non-terminating situations. Also, addressed a similar problem with an inventory model for non-instantaneously deteriorating items with nonlinear stock-dependent demand, hybrid payment scheme and partially backlogged shortages. Roy *et al.* (2022) analyzed a model for the facilities of advance payment and preservation technology investment and concurrent attempts to deal with shortages. Shah *et al.* (2023) established and formulated a general supply chain coordination with flexible payment policy under effect of green technology investments.

The third pertinent body of literature examines the deterioration of inventory, focusing on its impact on supply chain management. Ghare and Schrader (1963) pioneered the incorporation of deterioration effects in inventory systems. Benkherouf (1995) also proposed an inventory model incorporating deteriorating items, time-varying demand with a decreasing trend, and shortage considerations. Abad (2001) created an inventory model to help resellers optimize pricing and order quantities with partial backordering. Thereafter, Ouyang et al. (2006) investigated the optimal ordering strategy for deteriorating items with partial backlogging and permissible payment delays. Thereafter, Shah and Shah (2000), Li *et al*. (2010), Bakker *et al*. (2012), and Janssen *et al*. (2016) contributed to the evolving body of research on inventory systems for perishable items, offering updated analyses and insights. Zwaida *et al.'s* (2021) research addressed the drug refilling optimization problem, providing an integrated approach to hospital pharmacy inventory management. Kaushik (2023) analyzed EOQ models with and without preservation, focusing on Weibull deterioration and ramp-type demand.

The objective of this study is to determine the optimal selling price, order quantity, and retailer's profit, considering the impact of customer returns and advance payment to the supplier. For this, we consider single products with constant deterioration that are supplied by a single supplier. Hence two advance payment approaches are considered for suppliers for product orders: partial advance payment in equal installments; full partial prepayment. In real-world practice, prominent industries and online marketplaces, such as Amazon and Flipkart, provide flexible customer return policies, and advance payment is not a novel idea.

Table 1 presents a comprehensive comparative analysis of the extant literature, elucidating the research trajectory and highlighting existing research gaps.

Table 1. Comparative literature analysis and this study's contribution.

Author(s) name	Payment policy	Deterioration	Customer return
Ghare and Schrader (1963)	Cash on delivery	✓	✗
Pasternack (1985)	Cash on delivery	✓	✓
Abad (2001)	Cash on delivery	✓	✓
Chin *et al.* (2006)	Cash on delivery	✗	✓
Chen and Bell (2009)	Cash on delivery	✗	✓
Zia and Taleizadeh (2015)	Advance payment	✓	✗
Shah et al. (2015)	Trade credit	✓	✓
Nath et al. (2021)	Advance payment	✓	✗
Khan et al. (2022a)	Hybrid cash-Advance payment	✓	✗
This study	Advance payment	✓	✓

This paper addresses the research gap in the relationship between advance payment management, price-sensitive retail optimization, and consumer returns.

This paper is structured as follows: introduction that includes a brief summary of the literature and the research gap; presenting the notations and assumptions; formulating the problem mathematically; solving the problem; the numerical illustration; Sensitivity analysis is and conclusion with recommendations for further research.

2. Notations and Assumptions

2.1. Notations

Symbol	Description
A	Set-up cost ($\$ / order$)
C	Purchase cost ($\$ / unit$)
h	Holding cost ($\$ / unit / time$)

γ	Percentage of the purchase cost for the prepayment
θ	Constant rate ($\$/unit$) of deterioration for product
L	Allowed time duration to accomplish the prepayment
I_C	Anum interest charged (%)
T	Cycle time ($year$) (Decision variables)
P	Selling price ($\$/unit$), $P > C$ (Decision variables)
$\Pi_i(P,T)$	The retailer's average total profit (in \$), $i = 1,2$ (Objective function)

Expression and functions

$D(P,t)$	Demand rate ($\$/unit$) for product
$CR(P,t)$	Customer return rate ($\$/unit$) for product
$I(t)$	Inventory level ($unit$) at time $0 \le t \le T$ for product
Q	Ordering quantity per cycle ($unit$)

2.2. Assumptions

The following assumptions are considered to develop models:

1. Single product is considered.
2. Lead time is zero and shortages are not allowed.
3. The planning horizon is infinite.
4. Customer demand rate $D(P,t) = aP^{-\eta}(1 + bt - ct^2)$ where scale demand $a > 0$, linear rate of change of demand $0 \le b < 1$, quadratic rate of change of demand $0 \le c < 1$, price elasticity $\eta > 0$ (Shah et al. (2015)).
5. Customer return demand rate $CR(P,t) = \alpha D(P,t) + \beta P$ where $0 \le \beta, 0 \le \alpha < 1$ (Chen and Bell (2009)).
6. Return shipping, repacking, chargeback fee deteriorating items during the period under are not considered.

7. The initial and final inventory levels both are zero.
8. Products deteriorate at a constant rate θ ($0<\theta<1$). The deteriorated products have no salvage value.

3. Mathematical model

$I(t)$ the product stock level of product at any time at t with due to deterioration constant rate θ per time and customers demand $D(P,t)$ per time starts decline at time over the period $[0,T]$ can be represented by the following differential equation.

$$\frac{dI(t)}{dt}+\theta I(t)=D(P,t),0\le t\le T \tag{1}$$

Since product runs stocks out at $t=T$ (i.e. $I(T)=0$)

The solution of equation (1) using is given by

$$I(t)=\frac{aP^{-\eta}}{\theta^3}\begin{pmatrix}\left(e^{\theta(T-t)}\left(1+bT-cT^2\right)\theta^2+\left(2cT-b\right)\theta-2c\right)\\-\left(\left(1+bt-ct^2\right)\theta^2+\left(2ct-b\right)\theta-2c\right)\end{pmatrix} \tag{2}$$

Starting Q is on hand inventory of new product (i.e. $I(0)=Q$)

Now using $I(0)=Q$ from equation (2) gives

$$Q=\frac{aP^{-\eta}}{\theta^3}\left(\left(e^{\theta T}\left(1+bT-cT^2\right)\theta^2+\left(2cT-b\right)\theta-2c\right)-\left(\theta^2-b\theta-2c\right)\right) \tag{3}$$

3.1. Partial advance payment in equal installment

In such a case, we develop an inventory model for retailers who make advance partial payments in equal installments, as offered by suppliers. The retailer pays γ % of the total purchase price to the supplier L months in advance with equal N instalment. Thus, retailer’s partial advance payment (γCQ) has to be financed at the interest rate I_C and the remaining portion $(1-\gamma)CQ$ of total purchase cost is to be paid at the time of receipt of the ordered products, the inventory depletes as shown in Figure 1.

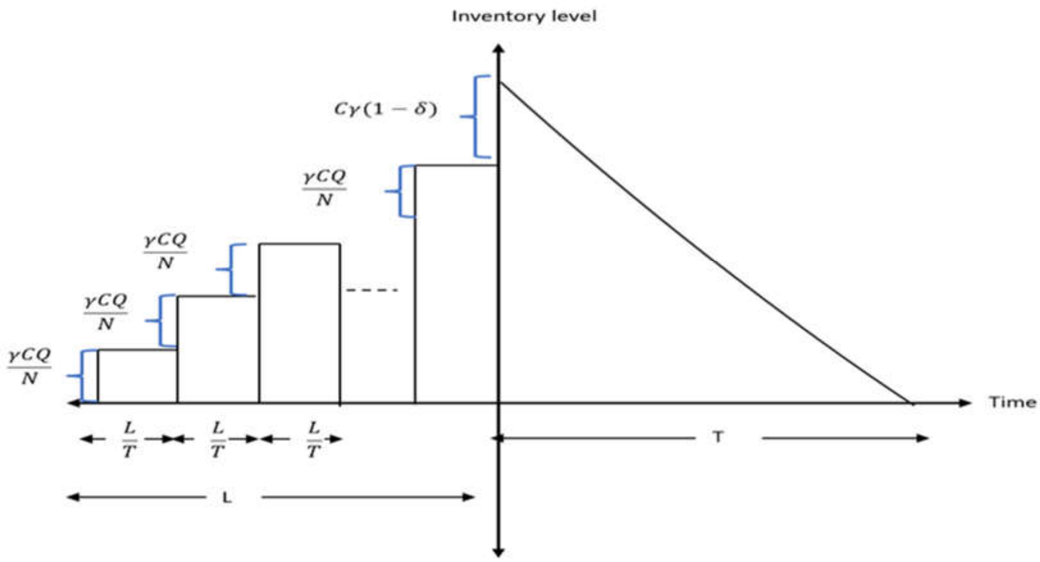

Figure 1. Graphical representation of inventory level under partial advance in equal installment.

The retailer's revenue and cost consist of the following components.

Retail sales revenue represents the income accrued by retailers from product sales.

$$SR = P\int_0^T D(P,t)dt \tag{4}$$

Purchase Cost

$$PC = \gamma CQ + (1-\gamma)CQ \tag{5}$$

Holding cost

$$HC = h\int_0^T I(t)dt \tag{6}$$

Ordering cost

$$OC = A \tag{7}$$

Return cost occurs when a customer returns a product to a retailer without including return charge of shipping, repacking, chargeback fee.

$$RC = P\int_0^T CD(P,t)dt \tag{8}$$

Interest cost for borrowing money from market to pay advance payment.

$$ICB = \frac{I_C \gamma CQL}{N}\left(1+2+..+N\right) \tag{9}$$

Therefore, retailer the total average profit per time is given by

$$\Pi_1\left(P,T\right) = \frac{1}{T}\left(SR - PC - HC - OC - RC - ICB\right) \tag{10}$$

$$\Pi_1\left(P,T\right) = \frac{1}{T}\left(P\int_0^T D(P,t)dt - CQ - h\int_0^T I(t)dt - A - P\int_0^T CD(P,t)dt - \frac{I_C \gamma CQ\left(N+1\right)}{2N}\right) \tag{11}$$

3.2. Full partial advance payment

The retailer pays $\gamma\%$ of the total purchase price to the supplier L months in advance. Thus, retailer's advance payment γCQ has to be financed at the interest rate I_C. The inventory depletes as shown in Figure 2.

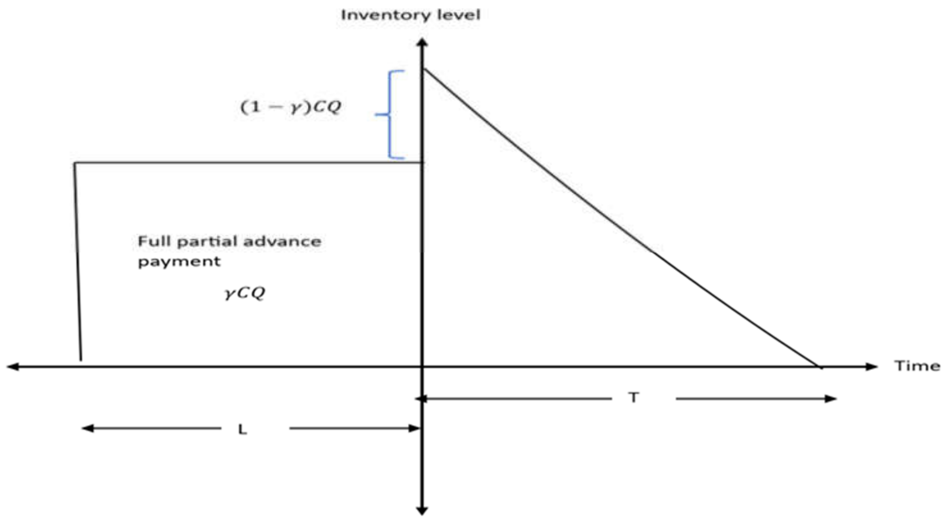

Figure 2. Graphical representation of inventory level under full partial advance.

Sales revenue, purchase cost, holding cost, ordering cost and return cost are considering as same as *3.1*

Interest cost of borrowing money from market to pay advance payment.

$$ICB_1 = I_C L\gamma CQ \tag{12}$$

Therefore, retailer the total average profit per time is given by

$$\Pi_2(P,T) = \frac{1}{T}\left(SR - PC - HC - OC - RC - ICB_1\right) \tag{13}$$

$$\Pi_2(P,T) = \frac{1}{T}\left(P\int_0^T D(P,t)dt - CQ - h\int_0^T I(t)dt - A - P\int_0^T CD(P,t)dt - I_C L\gamma CQ \right) \tag{14}$$

4. Solution procedure

To achieve optimality, the objective functions (11) and (14) must satisfy the following necessary condition.

$$\frac{\partial \Pi_i}{\partial P} = 0, \frac{\partial \Pi_i}{\partial T} = 0 \quad \text{for} \quad i = 1,2 \tag{15}$$

The solution procedure adopted is as follows.

- Step 1: Assign values to all inventory parameters in Equations (11) and (14).
- Step 2: Now solving Equation (15) simultaneously by mathematical software Maple 18
- Step 3**:** Check sufficient condition for $i = 1,2$

i.e. $\Delta_{11} = \frac{\partial^2 \Pi_i}{\partial P^2} < 0$, $\Delta_{22} = \frac{\partial^2 \Pi_i}{\partial T^2} < 0$, and $D = \det\left(\begin{bmatrix} \frac{\partial^2 \Pi_i}{\partial P^2} & \frac{\partial^2 \Pi_i}{\partial P \partial T} \\ \frac{\partial^2 \Pi_i}{\partial T \partial P} & \frac{\partial^2 \Pi_i}{\partial T^2} \end{bmatrix}\right) > 0$

Otherwise, different value in Step 1.

- Step 4: Optimal solutions P^* and T^* from Step 2 for Equations (11) and (14).
- Step 5: Obtains retailer's total profit using Equations (11), (15), and optimal ordering quantity of product $Q^*(unit)$ from Equation (3).
- Step6: Stop

The proposed models are illustrated below by considering the following two examples.

5. Numerical illustration

The benefits and the real-world impacts of the suggested framework are exemplified by the following numerical examples.

Example 1: The following input parameters in proper unit are considered for Case I.

$a = 10000\ units$, $b = 5\%$, $c = 10\%$, $\eta = 1.1$, $\alpha = 0.1$, $\beta = 0.4$, $C = \$20\ per\ unit$, $A = \$\ 200\ per\ order$, $h = \$\ 4/unit/year$, $\theta = 0.1$, $I_C = 0.12\%/\$\ /year$, $\gamma = 50\%$, $L = 1.5\ year$, $N = 5$

Example 2: The following input parameters in proper unit are considered for Case II.

$a = 10000\ units$, $b = 5\%$, $c = 10\%$, $\eta = 1.1$, $\alpha = 0.1$, $\beta = 0.4$, $C = \$20\ per\ unit$, $A = \$\ 200\ per\ order$, $h = \$\ 4/unit/year$, $\theta = 0.1$, $I_C = 0.12\%/\$\ /year$, $\gamma = 50\%$, $L = 1.5\ year$

Table 1. Optimal solution for Example 1 and 2.

Example	$P^*(\$)$	$T^*(year)$	$Q^*(unit)$	$\Pi_i^*(P,T)$ $(\$)$	Δ_{11}	Δ_{22}	D
1	59.32	0.68	78.51	1185.05	-2.65	-1517.45	3984.62
2	56.23	0.67	82.72	1615.26	-2.67	-1521.71	4520.65

6. Sensitivity analysis

A sensitivity analysis is conducted for Example 1, examining the impact of inventory parameter variations (-20%, -10%, +10%, +20%) on optimal solutions. The impact of parameter variations on decision variables and retailer's total average profit is summarized in following Figure 3-6.

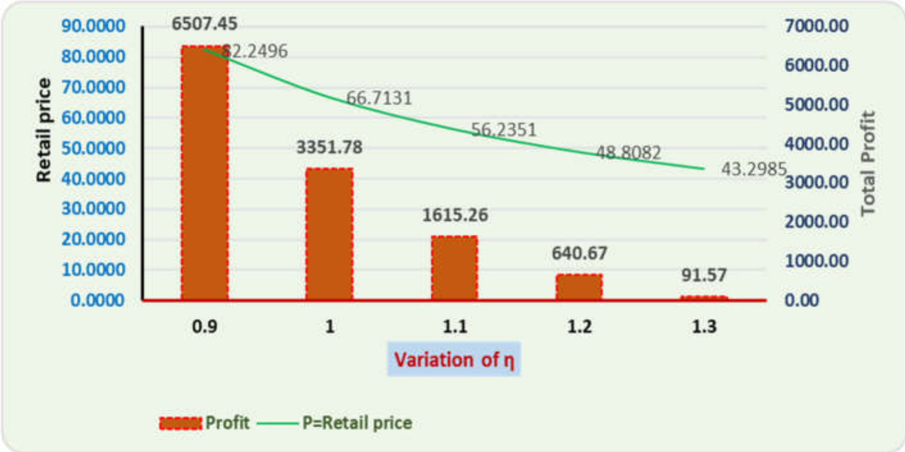

Figure 3. Variation in total profit with respect to η.

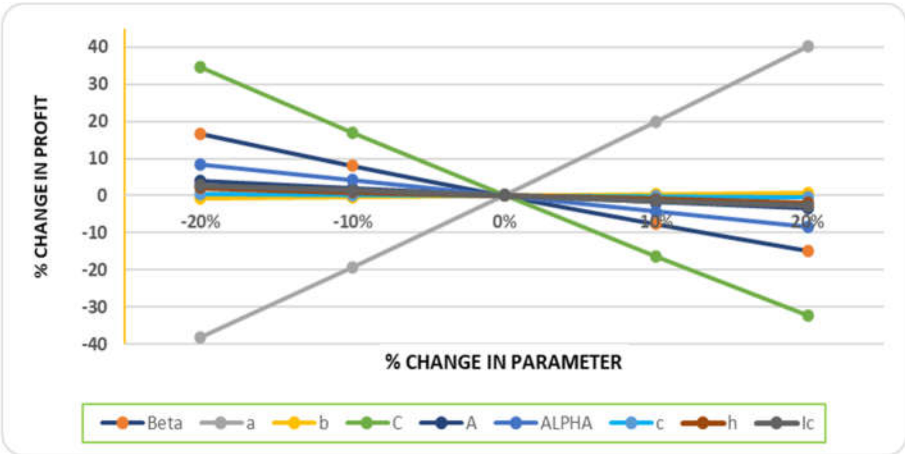

Figure 4. Percentage variation in total profit with respect to inventory parameters.

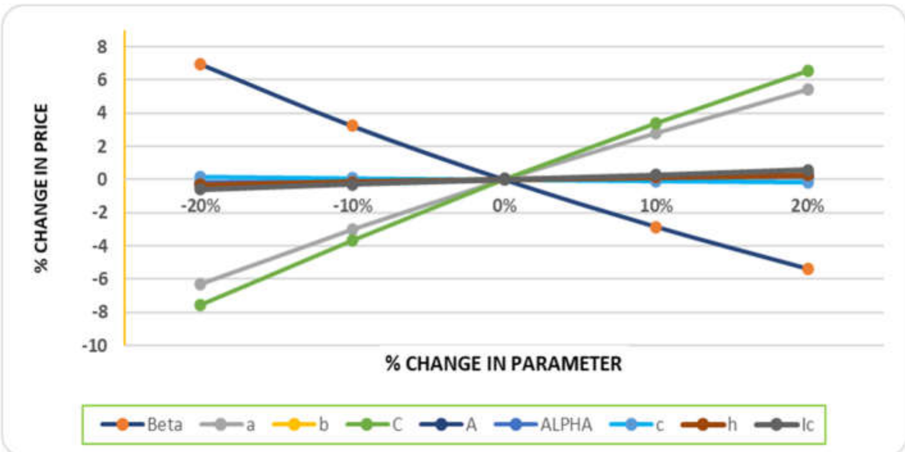

Figure 5. Percentage variation in selling price with respect to inventory parameters.

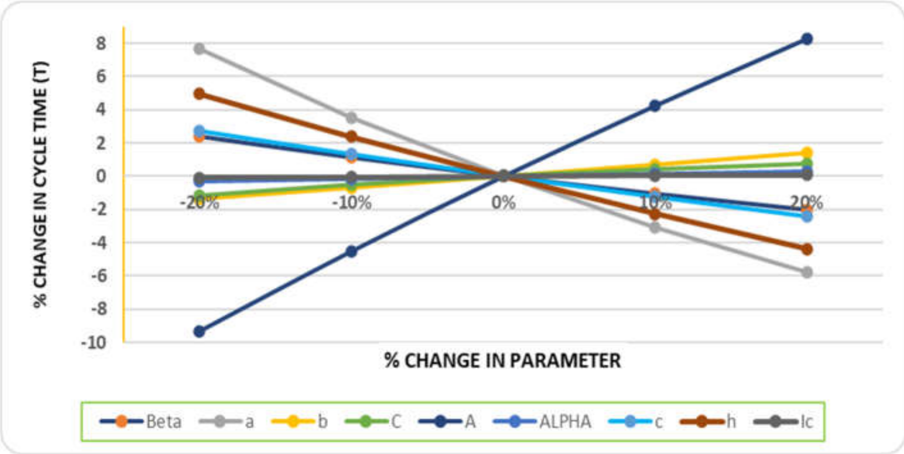

Figure 6. Percentage variation in cycle time with respect to inventory parameters.

Observation and actionable recommendations

From Figure 3-6 the following points are observed.

- Figure 3 reveals that price sensitivity (η) has a profound negative effect on the retailer's total average profit, selling price. If 18.18 % decreasing the values of η that increasing 275.49 % retailer's total average profit. This implies that demand responds significantly to price change.
- Figure 4 illustrates that parameter (a) and (b) have a substantial positive impact on profit, whereas parameters $\beta, C, c, A, h, \alpha$, and I_C exhibit minimal adverse effects. the study provides actionable recommendations for managers to optimize scale demand drivers, including customer satisfaction, product quality, and other critical factors.
- Figure 5 identifies (C) and (a) as critical factors boosting selling price, while (β) hinders price growth. Parameters $\alpha, b, A, h,$ and I_C have limited positive effects, and (c) has a slight negative effect. This implies that if the retailer can effectively integrate advance payment and return, it will have less effect on the selling price. Furthermore, the retailer should stock when the purchase price is low.
- Figure 6 highlights ordering cost (A) as a critical factor in reducing retailer's cycle time. Parameters $\alpha, b, C,$ and I_C have modest positive impacts, while $a, \beta, c,$ and h increase cycle time. The study provides actionable recommendations for retailers to achieve efficient cycle time reduction by adopting environmentally friendly ordering and cost-control measures.

7. Conclusions

This research investigates the synergistic effects of integrating advance payment, customer returns, and retail inventory management strategies to optimize revenue in price-sensitive consumer behavior. By integrating advance payment, customer returns, and retail inventory management, retailers can unlock a threefold advantage: improved supplier relationships, enhanced customer experience, and increased earnings. Research shows that full partial advance payments indicate a significant 36% profit growth relative to partial advance payments with full equal installments.

7.1. Unique contributions of study and future research directions

The numerical example demonstrates the effectiveness of the proposed model as a decision-making tool for retailers. Here are some future research directions: Exploring digital payment solutions, advance-cash-credit payment structures, and trade credit options; Sensitivity analysis to examine the impact of parameter changes; Extending the model to account for real-world complexities.

7.2. Limitations and challenges

Effective customer relationship management along with maintain advance payments to suppliers to order stock is a bit a challenge for the retailers such as complexity in supply chain management, difficulty in predicting customer behavior, and deterioration or obsolete products.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

None declared

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