A Review on Fuzzy Linear Programming Problem

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Abstract

In the present paper a review work is done in fuzzy linear programming problems. The basic definitions and some literatures of previously done work are discussed. The work is purely based on a collection of definitions and methods from known sources.

Key words: Linear programming, Fuzzy set, Operation research

1.1 Introduction:

A contemporary method for using mathematics to solve managerial problems is operations research (OR). The limits of operation research have not yet been established because it addresses problems in numerous fields. The ability to manage challenging problems is hence the formal definition of the phrase "operational research." When a choice is validated using mathematical and quantitative methods, the OR starts. Making the right decisions for the improvement of organizations is one of a manager's main responsibilities in the business sector. Every human being makes a variety of decisions in their daily lives without even realizing it. In uncomplicated scenarios, these choices are determined by intuition, discernment, and expertise without relying on any form of mathematical or other models. It is widely understood that the decision-making process is intricate and crucial in our daily lives, but it becomes even more challenging for a manager as they are aware that their decisions directly impact the organization's growth. For instance, designing a public transportation system in a city with its unique arrangement of factories and residential areas, or determining the optimal product mix when there are numerous products with varying profit contributions and production needs.

Operational Research is not confined to any particular field. It draws on various disciplines including mathematics, statistics, economics, management psychology, engineering, and others, to integrate these tools and create new knowledge for effective decision-making. Today, O.R. has evolved into a specialized area that applies scientific methods to decision-making, especially in the efficient allocation of resources. It aims to offer a logical foundation for decision-making when complete information is not available, as human beings, machines, and procedural systems may lack comprehensive information.

In the contemporary age, operations research can also be viewed as a scientific field, as it encompasses the study of problems and the anticipation of system behavior, particularly in human-machine systems. therefore, OR expert engages in three classical aspects of science:

i) Determining the behavior of a system
ii) Analyzing the behavior of a system by developing appropriate models
iii) Predicting future behavior using such models

Operations research (OR) are a multidisciplinary field that originated during World War II to address various problems. Today, it offers improved solutions for business applications, particularly in the realm of operating systems for evaluating different courses of action. Linear programming models are highly effective in OR for resolving a wide range of issues, such as profit maximization, cost minimization in transportation, labor, and production, job scheduling, and more.

1.2 Linear Programming Problems

Linear programming is a valuable and flexible method within the field of Operations Research that can be utilized for various managerial challenges such as analyzing transportation dilemmas, distribution strategies, production optimization, investment decisions, oil refining processes, and more. This technique is not only beneficial for resolving issues within industries and businesses, but it also proves to be advantageous in non-profit sectors like education, government agencies, hospitals, libraries, and beyond. The application of linear programming is particularly suitable for problems that involve decision variables. The limitations and target function of linear programming issues can be expressed as a linear portrayal of the choice variable. The effectiveness of the system is quantified by a primary objective standard or aim, such as boosting profits or efficiency or reducing costs and usage. There are constantly real-life restrictions on the supply of various resources, such as the accessibility of personnel, raw materials for manufacturing, vehicles for transport, or time restrictions within the system. These constraints are laid out as linear equations along with decision variables. To resolve a linear programming issue, we compute the specific values of the decision variables that can maximize the objective function while adhering to the specified constraints.

The characteristic of linearity is crucial in the linear programming model, which has numerous applications in both profitable and non-profitable organizations. While some linear programming models may not be strictly linear, they can be transformed into linear form through mathematical adjustments. Likewise, many real-world applications may not be linear, but they can still be effectively analyzed using linear programming models. Due to their ease of solving, linear programming models are often utilized to address nonlinear issues that would otherwise be difficult to solve.

1.2.1 Mathematical Formulation of Linear Programming Problem:

L.V. Kantrovich, a Russian mathematician, utilized a mathematical model to address the linear programming problem. In 1939, he emphasized that numerous production-related problems can be mathematically defined and subsequently solved numerically. Subsequently, G.B. Danzing[18] presented a paper on the linear programming problem in 1948, titled "Linear Programming," which played a significant role in the future numerical analysis of such problems, contributing to the development of the linear programming problem. Now, we discussed the fundamental characteristics of any optimization problem:

- **Variables:** When we start to solve our problems, at that time we don't know about the value of variables. Normally, variables represent things that you want to adjust or control in our problems, for example, the cost of manufactured items, rate of transportation, etc. The motive is to find values of the variables that provide the best value of the objective function.
- **Objective Function:** Objective function is a mathematical presentation that combines the variables to present your goal. It may represent profit, for example. You will want to either maximize or minimize the objective function.
- **Constraints:** Constraints define the boundaries of potential solutions through mathematical representations of variables. For instance, they may indicate that there are a finite number of workers available to operate a specific machine, or that only a specific quantity of steel can be provided each day.
- **Variable Bounds:** Only rarely are the variables in an optimization problem permitted to take on any value from minus infinity to plus infinity. Instead, the variables usually have bounds. For example, zero and 1000 might bound the production rate of widgets on a particular machine.

So, Linear Programming can be shown with *n* decision variables and *m* constraints in terms of mathematics in the following manner

Objective function (Min or Max) $Z = c_1x_1 + c_2x_2 + c_3x_3 \cdots c_nx_n$

Subject to:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \cdots a_{1n}x_n (\leq, =, \geq) b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \cdots a_{2n}x_n (\leq, =, \geq) b_2$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 \cdots a_{mn}x_n (\leq, =, \geq) b_m$$
$$\text{and } x_1, x_2, x_3, \cdots x_n \geq 0$$

The mathematical expression can change into compact form by using the summation symbol as follows:

Optimize (Min or Max) $Z = \sum_{j=1}^{n} c_jx_j$ (1.1)

Subject to: $\sum_{j=1}^{n} a_{ij}x_j \ (\leq, =, \geq) b_i \ i = 1,2,\cdots,m$ (1.2)

and $x_j \geq 0 \quad j = 1,2,\cdots,n$ (1.3)

equation (1.1) is known as the Objective function, similarly equations (1.2) and (1.3) are known as constraints and non-negative conditions respectively.

Where,

Z = Objective function of our modal

x_j = Decision variables of the objective function

c_j = Coefficients representing the per unit contribution of the decision variable

b_i = availability at the i^{th} resource

a_{ij} = input-output coefficient and it shows the number of resources.

1.2.2 Some Important Definitions of Linear Programming Problems:

(a) Objective function:

The objective function minimizes and maximizes the cost and profit respectively of our LP modal.

(b) Constraints:

Constraints stipulate the conditions under which a feasible solution can be derived from the variables that make up the decision.

(c) Non-Negative:

In every linear programming model, each decision variable should be greater than or equal to zero, regardless of whether the goal is to maximize or minimize the net present value of an activity.

(d) Solution:

$x_1, x_2, x_3 \cdots x_n$ are real numbers that satisfy the constraints of a Linear Programming Problem and are called the solution of a Linear Programming Problem.

(e) Infeasible Solution:

It is said that an infeasible solution to the LP model will consist of x_j $(j = 1, 2, \cdots n)$ values that satisfy none of the constraints and non-negativity conditions.

(f) Feasible:

It is said that a feasible solution to the LP model will consist of x_j $(j = 1, 2, \cdots n)$ values that satisfy all the constraints and non-negativity conditions.

(g) Basic Solution:

A basic solution is achieved by setting (n-m) variables to zero and then solving the remaining m equations in m variables for a set of m simultaneous equations with n variables (n>m).

(h) Basic feasible solution: A linear programming problem that possesses a basic feasible solution is referred to as a basic feasible problem. In this context, it is assumed that all basic variables are non-negative. There exist two categories of basic feasible solutions.

(i) Degenerate: At least one basic variable has zero value, then the basic feasible solution is called degenerate**.**

(j) Non-degenerate: If the values of basic variables are positive and non-zero, then the basic feasible solution is known as non-degenerate.

(k) **Optimum Solution:** Optimal basic feasible solutions are those that maximize the objective function values of linear programming problems.

(l) **Unbounded solution:** An unbounded solution occurs when the objective function either decreases or increases indefinitely.

1.3 Fuzzy Set Theory:

The fuzzy set theory was proposed by L.A. Zadeh (1965) [66] and has been found extensive in various fields. Bellman and Zadeh (1970) [6] were the first to consider the application of the fuzzy set theory in solving optimization problems in a fuzzy environment. According to Zadeh fuzzy set is an extension of the classical notion of a set. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition "An element either belongs to or does not belong to the set".

We use fuzzy when we talk about real world, expressions of the real world, where we quantify the real world, is the way we describe the real world. They are not very precise, for example –words like young, tall, good, or height are fuzzy. For some people, age 25 is young and for others age 35 is young. The concept is that the young have no clear boundaries. Age 35 has some possibility of being young and usually depends on the context in which it is being considered. Similarly, when we talk about the height of any person, sometimes we say you are very tall because we cannot expect that his actual height is 5'9" similarly we cannot define what the actual temperature of today is. Above expression about real world are not always precise and the condition which not be precise is exactly fuzzy.

Fuzzy set: A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to the unit interval [0, 1] i.e. $A = \{(x, \mu_A(x); x \in X\}$, Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of xε X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

1.3.1 Some Basic Definitions of Fuzzy Set:

(a) A fuzzy set is defined by a membership function that maps elements of a domain, the universe of discourse X, to the unit interval [0, 1]. In other words, A = {(x, μA(x); x ∈ X}. Here, μA: X →[0,1] is a mapping known as the degree of membership function of the fuzzy set A, and μA(x) is referred to as the membership value of x∈ X in the fuzzy set A. These membership grades are commonly expressed as real numbers within the range of [0, 1].

(b) A fuzzy set consists of a pair (A, m) where A represents a set and m: A→ [0, 1]. The grade of membership of x in (A, m) is denoted by m(x) for each x ∈ A. In the case of a finite set A={x1,x2,x3,……..xn}, the fuzzy set (A, m) is typically represented as {m(x1)/x1, m(x2)/x2,………m(xn)/xn}. If x ∈ A, it is considered not included in the fuzzy set (A, m). A fully included element is one where m(x) = 0, while a fuzzy member is an element where m(x) = 1. For values of m(x) between 0 and 1, the set {x ∈ A; m(x)>0} is known as the support of (A, m), and the set {x ∈ A; m(x) = 1} is referred to as its kernel.

(c) The characteristic function μ_A of a crisp set A⊆X assigns a value of either 0 or 1 to each member in X. This function can be extended to a function μ_A such that the value assigned to the element of the universal set X falls within a specified range [0,1] i. e. $\mu_{\tilde{A}}$:X→ [0,1]. The assigned values indicate the membership grade of the element in the set A.

The function $\mu_A$is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x): x \in X)\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set. $\mu_{\tilde{A}}(X)$ is the degree of membership of x in$\tilde{A}$. The closer the value of $\mu_{\tilde{A}}(X)$ is to 1, the more x belongs to A.

1.3.2 Membership Function

Membership functions were introduced by Zadeh [66]in 1965 and defined that a fuzzy set is completely characterized by its membership function (M F). Since most fuzzy sets in use have a universe of discourse X consisting of the real line R, it would be impractical to list all the pairs defining a membership function. A more convenient and concise way to define M F is to express it by a mathematical formula which is given in 1.3.1(a) and 1.3.1(b). Sometimes a more general definition is used, where the membership function takes values in an arbitrary fixed algebra or L, known as L – Fuzzy Sets, described by J. A. Goguen (1967) [30].

Membership functions can either be chosen by the user arbitrarily, or based on the user's experience (MF chosen by two users could be different depending upon their experiences, perspectives, etc.) Or be designed using machine learning methods (e.g., artificial neural networks, genetic algorithms, etc.). There are different shapes of membership functions; triangular, trapezoidal, piecewise-linear, Gaussian, bell-shaped, etc. In the chapters, we will discuss about the membership functions of triangular or trapezoidal fuzzy numbers. The graphical representation of membership functions are:

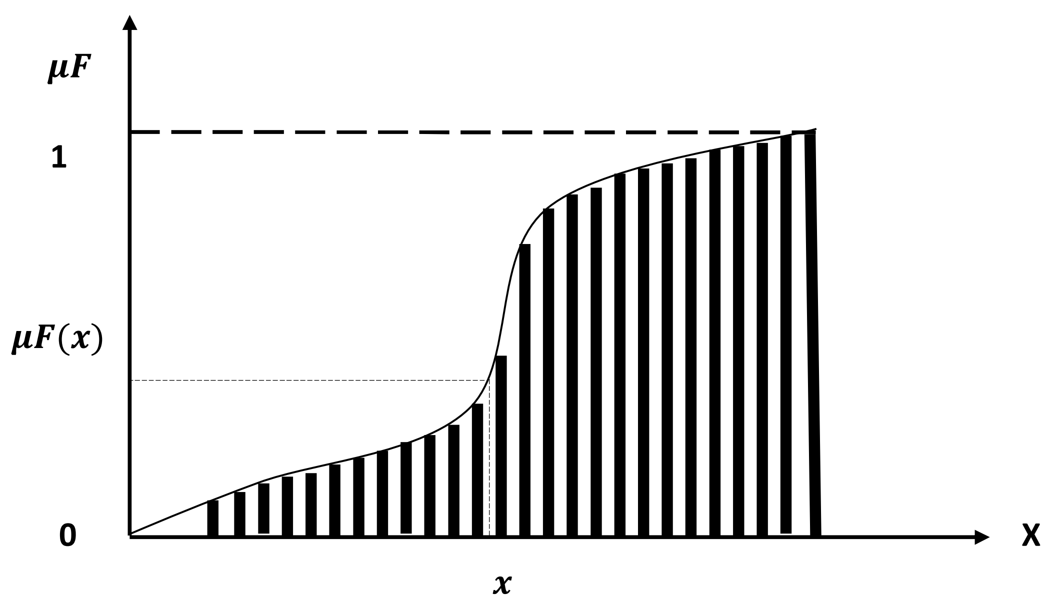

Figure: 1.1 Membership Function of a Fuzzy Set

1.3.3 Fuzzy Numbers

The theory of fuzzy numbers, as established by Dubois and Prade [21] in 1980, introduces a limited class of Membership Functions. Fuzzy numbers are quantities that lack precision, unlike "ordinary" numbers which have exact values. A fuzzy number can be viewed as a function with a domain typically set as the real numbers, and a range spanning non-negative real numbers from 0 to 1. Each value in the domain is associated with a particular "grade of membership," ranging from 0 as the lowest grade to 1 as the highest grade.

Classical fuzzy numbers with contradictory properties are useful when it comes to a straightforward explanation in the language of set theory. Nevertheless, one might wonder: how can we visualize fuzzy data, like X, in a manner that combining it with another fuzzy data (number) results in a new fuzzy number C? Several scholars have attempted to address this query by introducing the concept of ordered fuzzy numbers, which can be represented by pairs of continuous functions defined on the interval [0, 1].

1.3.3.1 Types of Fuzzy Numbers

(a) Triangular Fuzzy Numbers

A number $\tilde{A}$ is a triangular fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3)$. where a_1, a_2, a_3 are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for a < x_1 \\ \dfrac{x - a_1}{a_2 - a_1} & for a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for a_2 \le x \le a_3 \\ 0 & for x > a_3 \end{cases}$$

By using min and max, we have an alternative expression for the proceeding equation:

triangle (x; a, b, c) = max $\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c},\right)\right)$

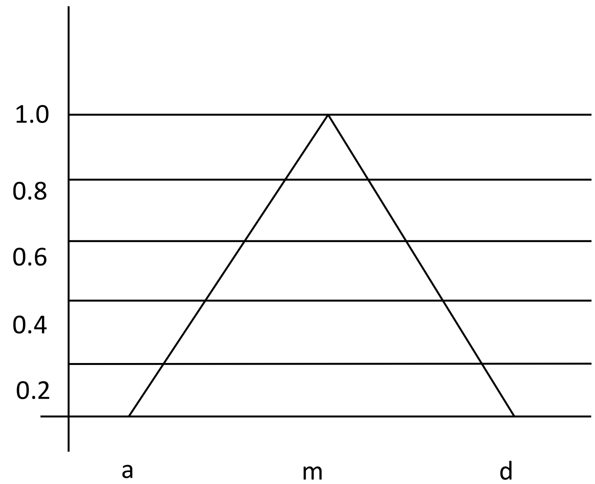

Figure: 1.2 Graphical Representation of Triangular Fuzzy Number

(b) Trapezoidal Fuzzy Numbers: A fuzzy number $\tilde{A}$ is a trapezoidal fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)$. where a_1, a_2, a_3, a_4 are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & for\ a_1 \leq x \leq a_2 \\ 1 & for\ a_2 \leq x \leq a_3 \\ \dfrac{a_3-x}{a_4-a_3} & for\ a_3 \leq x \leq a_4 \\ 0 & for\ x > a_4 \end{cases}$$

An alternative concise expression using min and max is:

Trapezoidal (x; a, b, c, d) = Max$\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c},\right)\right)$

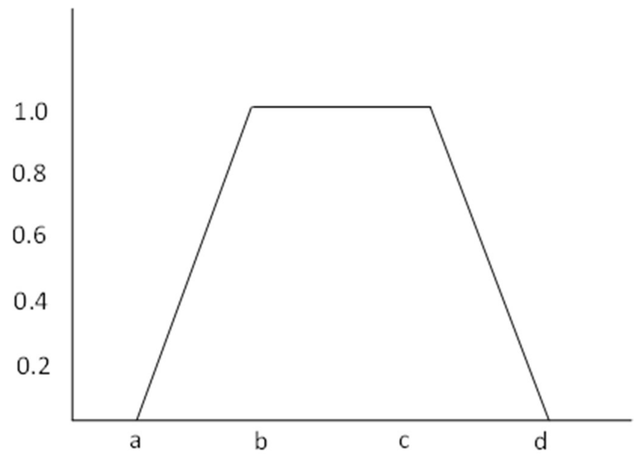

Figure 1.3: Graphical Representation of Trapezoidal Fuzzy Number

1.3.3.2 Operations of Fuzzy Numbers:

D. Dubois and H. Prade (1978) [21] describe some operations on fuzzy numbers. A few operations on triangular fuzzy numbers and trapezoidal fuzzy numbers from them are:

(a) Operations of Triangular Fuzzy Numbers

The following are the four operations that can be performed on triangular fuzzy numbers: let $\widetilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then,

- **Addition:** $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- **Subtraction:** $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
- **Multiplication:** $\tilde{A} \times \tilde{B} = (\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2,$

$$\max(a_1b_1, a_1b_3, a_3b_1, a_3b_3)$$

- **Division:** $\frac{\tilde{A}}{\tilde{B}} = (\min\left(\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}\right), \frac{a_2}{b_2}, \max(\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}))$

(b) Operations of Trapezoidal Fuzzy Numbers

The following are the four operations that can be performed on trapezoidal fuzzy numbers: let $\widetilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ then,

- **Addition:** $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

- **Subtraction:**$\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
- **Multiplication:**$\tilde{A} \times \tilde{B} = (t_1, t_2, t_3, t_4)$

  Where $t_1 = \min(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$

  $$t_2 = \min(a_2b_2, a_2b_3, a_3b_2, a_3b_3)$$

  $$t_3 = \max(a_2b_2, a_2b_3, a_3b_2, a_3b_3)$$

  $$t_4 = \max(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$$

- **Division:**$\frac{\tilde{A}}{\tilde{B}} = (\min\left(\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}\right), \min\left(\frac{a_2}{b_2}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_3}{b_3}\right),$

  $$\max\left(\frac{a_2}{b_2}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_3}{b_3}\right), \max\left(\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}\right))$$

1.4 Fuzzy Linear Programming Problem:

Linear Programming Problem is one of the most frequently applied operation research techniques. In the conventional approach value of the parameter of linear programming models must be well-defined and precise. However, in the real-world environment, this is not a realistic assumption. In real-life problems, there may exist uncertainty about the parameter. In such a situation the parameters of linear programming problems may be represented as fuzzy numbers. Fuzzy Linear programming problems with *m* fuzzy equality constraints and *n* fuzzy variables may be formulated as follows:

Optimize (Min or Max) $Z = \sum_{j=1}^{n} \tilde{c}_j x_j$ (1.4)

Subject to: $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \ (\leq, =, \geq) b_i;$ $i = 1,2,\cdots,m$ (1.5)

and $x_j \geq 0;\ ;$ $j = 1,2,\cdots,n$ (1.6)

Here,

$\tilde{c}_j$ = Objective Values in term of fuzzy number

x_j = Contribution per units

$\tilde{a}_{ij} =$ Input-output coefficient in term of fuzzy number

$\tilde{b}_i$ = Total availability of the i^{th} resource

1.5 Multi-Objective Optimization Technique

Many real-world optimization problems are multi-objective in nature and are to be optimized simultaneously subject to a common set of constraints. The most general mathematical model of multi-objective optimization problem is:

Maximize: $F(x) = [f_1(X), f_2(X), \dots \dots \dots, f_m(X)], X = (x_1, x_2, \dots \dots \dots x_n)$

Subject to

$$g_j(X) \leq 0, \qquad j = 1,2, \dots \dots., k$$

$$h_j(X) \leq 0, \qquad j = 1,2, \dots \dots., m$$

$$l_j(X) \leq 0, \qquad j = 1,2, \dots \dots., r$$

Where $f_1, f_2, \dots \dots.., f_m$ are the objective functions? Variables $x_1, x_2, \dots \dots.., x_n$ are called decision variables and X is called decision vector. This problem is also called a multi-objective programming problem.

1.5.1 Multi-Goal Fuzzy Linear Programming Problem (MGFLPP)

Multi-objective linear programming problem is a sub-section of a linear programming problem, which is used to fulfill one or more objectives. In this, our objectives are different – different but subject to constraints, and non-negative variables are considered to same for all objective functions.

So, the mathematical formulation of Linear Programming Problem (LPP)

Minimize: $\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}^r{}_{ij} x_{ij}$

Subject to constraint:

$\sum_{j=1}^{n} x_{ij} \leq \tilde{a}_i$ i=1, 2, ………..m

$\sum_{i=1}^{m} x_{ij} \geq \tilde{b}_j j$=1, 2,………..n

$x_{ij} \geq \quad 0$ for all i and j

Where,

$\tilde{c}^r$ = Multi-Objective function of linear programming problems

r = Number of objectives

1.6 Assignment Problems:

Assignment problems (AP) are special cases of linear programming problems. The task is to assign 'i' resources ($i = 1, 2, 3, 4 \dots m$) to 'j' tasks ($j = 1, 2, 3, 4 \dots n$) in such a way that the total cost of performing the tasks is lowest. The cost associated with performing resource-i by task-j is denoted by 'Cij'. Moreover, each resource should be exactly performed by one task. Thus, the assignment problem is a special type of Linear Programming Problem, where assignees are assigned to perform tasks. In general, assigning people to jobs is a common application; however, the assignees need not be people; they can also be machines, projects, locations, or plants. Classic examples, like assigning the machine operator to the most suitable job, assigning the set of project managers to the most suitable projects, and assigning the subject to suitable faculty typically fall under the category of assignment problems.

In the machine operator- job assignment, the objective of the firm may be reducing the overall time taken or reducing the wastages by appropriately assigning the operators to the suitable job; in assigning the project managers to suitable projects, the firm may have an objective of reducing the overall lead time or cost of operation; in the example of subject-faculty allocation, the university education administrator may be interested in reducing the total number of 'failures' in a class.

1.6.1 Assignment Model:

Consider a problem of assignment of n resource to n activities so as to minimize the overall cost or time in such a way that each source can be associate with one and only one job[1,2,3,4]. The effectiveness matrix is given as under:

	Activity					Available
Resource		P_1	P_2	⋯	P_n	
	S_1	T_{11}	T_{12}	⋯	T_{1n}	1
	S_2	T_{21}	T_{22}	⋯	T_{2n}	1
	⋮	⋮	⋮	⋮	⋮	⋮
	S_n	T_{n1}	T_{n2}	⋯	T_{nn}	1
Required		1	1	⋯	1	

1.6.2 Mathematical Formulation of Assignment Problem:

Let x_{ij} denote the assignment of facility i to j such that

$$x_{ij} = \begin{cases} 1 & if\ facility\ i\ is\ assigned\ to\ job\ j \\ 0 & otherwise \end{cases}$$

Then, the mathematical model of the assignment problem can be stated as:

$$\text{Min} \qquad Z = \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij}\, x_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for all i (recourse availability)}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \text{ for all j (activity requirement)}$$

and $$x_{ij} = 0\ or\ 1 \quad \text{for all i and j}$$

Where c_{ij} represents the cost of assignment of recourse i to activity j.

1.7 Methods for Solving Linear Programming Problem:

An optimization technique based on linear programming aims to optimize some criterion within constraints. Among other things, it includes an objective function that measures efficiency like profits, losses, and returns on investments, as well as several boundary conditions that restrict how the resources are used. There are several processors available to resolve a linear programming problem [1,2,3,4].

1.7.1 The Simplex Algorithm:

To find the solution of any type L.P.P. with the help of a simplex algorithm, we always assumed that an initial basic feasible solution exists. The steps for the computation of an optimum solution are as follows:

Step 1. Check whether the objective function of the given LPP is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximizing it by using the result

$$Minimum\ z = -\ Maximum\ (-z)$$

Step 2. Here, we check all values of b_i ($i = 1, 2, \ldots m$) are non-negative. If the value of any b_i is negative then we multiply with-1 respective inequation of the constraints, so we get all b_i ($i = 1, 2, \ldots m$) are non-negative.

Step 3. Introducing slack/surplus variables to change all the inequations of the constraints into equations. Here we have the value of variables equal to zero.

Step 4. To find an initial basic feasible solution to LP problem in the form$x_B = B^{-1}b$ and put it in the first column of the simple table.

Step 5. Compute the net evaluations $z_j - c_j$ ($j = 1, 2, \ldots n$) by using the relation $z_j - c_j = c_B y_j - c_j$ where $y_j = B^{-1}a_j$.

Inspect the sign $z_j - c_j$as follows

(1) If all $z_j - c_j \geq 0$ then the initial basic feasible solution x_B is an optimum basic feasible solution.

(ii) If at least one $z_j - c_j < 0$, proceed on to the next step.

Step 6. If there are more than one negative $z_j - c_j$, then choose the most negative of them. Let it be $z_r - c_r$, for some j = r.

i. If all $z_j - c_j \geq 0$ then the initial basic feasible solution x_B, is an optimum basic feasible solution.
ii. If at least one $z_j - c_j < 0$ proceed on to next step,

Step 7. Calculate the ratios $\left\{\frac{x_{Bi}}{y_{ir}}, y_{ir} > 0, i = 1,2,\cdots,m\right\}$ and pick the minimum of them. Let the minimum of these ratios be $\frac{x_{Bk}}{y_{kr}}$ Then the vector y_k will level the basis y_B. The common element y_{kr} which is in the k^th row and the r^th column is recognized as the leading element (or pivotal element) of the table.

Step 8. Change the leading element to unity by dividing its row by the leading element itself and all other elements in its column to zeroes by making use of the relations:

$$\widehat{y_{ij}} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir} i = 1,2,\cdots,m+1; i \neq k$$

and $$\widehat{y_{kj}} = \frac{y_{kj}}{y_{kr}} \qquad i = 0,1,2,\cdots,n$$

Step 9. Go to Step 5 and replication the solving procedure until either an optimum solution is obtained or there is an indication of an unbounded solution.

1.7.2 Big – M Method (Artificial Variable Technique)

The Big-M Method is an additional method of eliminating artificial variables from the basis. In this method, we allocate large undesirable (unacceptable penalty) coefficients to artificial variables from the objective function point of view. If objective function Z is to be minimized, then a very large positive price (called a penalty) is assigned to each artificial variable. Similarly, if Z is to be maximized, then a very large negative price (also called a penalty) is assigned to each of these variables. The penalty will be designated by –M for a maximization problem and +M for a minimization problem, where M > 0.

Computational Procedure of Big-M Method

Step I: Introduce the LP problem into the standard form by adding slack variables, surplus variables,   and artificial variables.

Step II: Presenting non-negative variable to the left side of all the constraints of $(\geq or =)$ type. These variables are called artificial variables. The purpose of introducing artificial variables is just to obtain an initial basic feasible solution.
Step III: Solve the Modified LPP by simplex method, until any one of the three cases may arise.

(i) If no artificial variable appears in the basis and optimality conditions are satisfied, then the current solution is an optimal basic feasible solution.

(ii) If at least one artificial variable in the basis at zero level and the optimality condition is satisfied then the current solution is an optimal basic feasible solution.

(iii) If at least one artificial variable appears in the basis at the positive level and the optimality condition is satisfied, then the original problem has no feasible solution.

1.8 Elementary Transformation Method:

Step 1: Construct of LPP**:**

Min or Max $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} . x_{ij}$

Subject to Constraints

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \ (\leq, =, \geq) \ b_j, \quad i = 1, 2, \ldots, m$$

$$x_j \quad \geq \quad 0, \quad j = 1, 2, \ldots, n$$

Step 2: Subject to constraints considered as a system of linear equations:

$$a_{11}x_1 + b_{12}x_2 + c_{13}x_3 = d_1$$

$$a_{21}x_1 + b_{22}x_2 + c_{23}x_3 = d_2$$

$$a_{31}x_1 + b_{32}x_2 + c_{33}x_3 = d_3$$

Step 3: The system to the equation can be changed into the matrix form

$$\text{Coefficient Matrix } A = \begin{bmatrix} a_{11} & b_{12} & c_{12} \\ a_{21} & b_{22} & c_{22} \\ a_{31} & b_{32} & c_{33} \end{bmatrix} \text{ Constant Matrix } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\text{Variable Matrix } X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Step 4: Construct an augmented matrix with the help of coefficient and constant matrix i.e. [A: B].

Step 5: After solving an augmented matrix, we have got following two conditions:

Case I: Set of Linear equation is called inconsistent if Rank of A ≠ Rank of [A: B] then we have not got basic variables.

Case II: A set of Linear equations is called consistent if Rank of A = Rank of [A: B] then we have got basic variables if the rank of the augmented matrix is equal to the number of unknown variables

Step 6: To find the optimum solution, **t**he system of equations can be written as i.e. $AX = B$.

1.9 CHANDRA SEN'S METHOD [53]:

The Steps involved in the algorithm are:

Step 1: Apply the Big M Method and determine the optimum solution for every objective function.

Step 2: Let Max $P_n = \chi_k$ where k = 1,2,3 …g $P_n = \chi_k$ Where K = g+1, g+2…..h.

Step 3: Calculate B_1 and B_2 Where B_1 = max ($|\chi_k|$ where k = 1,2,3 …g and B_2 = min $|\chi_k|$ Where K = g+1, g+2…..h.

Step 4: Calculate the value of the Mean by Different Mean Method.

a) Arithmetic Mean Method:

Arithmetic Mean

$$Max\ P = \frac{\left(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n\right)}{A.M.}$$

Arithmetic Mean by Average

$$A.M. = \frac{B_1 + B_2}{2}$$

b) Quadratic Mean Method

Quadratic Mean

$$Max\ P = \frac{\left(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n\right)}{Q.M.}$$

Quadratic Mean by Average

$$QM = \sqrt{\frac{(B_1^2 + B_2^2)}{2}}$$

c) Geometric Mean Method:

Geometric Mean

$$Max\ P = \frac{\left(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n\right)}{G.M.}$$

Geometric Mean by Average

$$GM = \sqrt{B_{1\times}B_2}$$

d) Harmonic Mean Method:

Harmonic Mean

$$Max\ P = \frac{\left(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n\right)}{H.M.}$$

Harmonic Mean by Average

$$HM = \frac{2}{\frac{1}{B_1} + \frac{1}{B_2}}$$

e) Heronian Mean Method

Heronian Mean

$$Max\ P = \frac{\left(\sum_{n=1}^{g} P_n - \sum_{n=g+1}^{h} P_n\right)}{H_e.M.}$$

Heronian Mean by Average

$$H_e.M = \frac{1}{3}\left(B_1 + \sqrt{B_{1\times}B_2} + B_2\right)$$

1.10 Heuristic Method:

Step 1: Calculate the difference between the two lowest cost cells (called Penalty) for each row and column. These are called as row and column penalties, P, respectively.

Step 2: Add the cost of cells for each row and column. These summations are called row and column cost, T, respectively.

Step 3: Compute the product of penalty 'P' and the total cost 'T', that is PT for each row and column.

Step 4: Identify the row/column having the lowest 'PT'.

Step 5: Choose the cell having minimum cost in the row/column identified in Step 4.

Step 6: Make the maximum feasible allocation to the cell chosen in Step-5, if the cost of this cell is also minimum in its column/row. Otherwise, allocation is avoided and go to Step 7.

Step 7: Identify the row/column next to the lowest 'PT'.

Step 8: Choose the cell having minimum cost in the row/column identified in Step 7.

Step 9: Make the maximum feasible allocation to the cell chosen in Step 8.

Step 10: Cross out the satisfied row/column.

Step 11: Repeat the procedure until all the requirements are satisfied.

Literature Survey:

The concept of fuzzy set theory was first introduced in 1965 by L. A. It was introduced by Zadeh [68] and has since been widely applied in various fields. Bellman and Zadeh (1970) [9] were pioneers in exploring the use of fuzzy set theory to solve optimization problems in fuzzy environments. He emphasized that both the objective function and the constraints in the model can be represented by corresponding fuzzy sets and should be treated in the same manner. A. Kaufman (1976) [36] also contributed to the theory of fuzzy sets by introducing some of its properties. In 1978, Zadeh [69] further developed the concept of fuzziness and presented a theory on the possibility of fuzzy sets to quantitatively handle ambiguous information in decision-making.

To tackle linear programming and fuzzy programming problems with multiple objective functions, Zimmermann (1978) [70] used the fuzzy set theory idea with some appropriate membership functions. Maximizing profits and fuzzy transportation difficulties can be solved with appropriate modifications to transportation networks by addressing the fuzzy transportation problem. The issue initially appears in the form of linear programming with multiple objective functions and fuzzy programming as described by H. J. Zimmermann (1978) [70], Prade (1980) [45], and others. M OL h EL Igeartaigh (1982) [31], Chanas (1984) [14], and others have highlighted the fuzzy approach as a solution to the transportation problem. These studies demonstrate the application of this approach through simple examples based on real-life scenarios. However, it is important to note that these models focus on single objectives and traditional two-index transportation issues. In practice, transportation problems often involve multiple objective functions such as average delivery time and minimizing costs. H Tanaka (1984) [58] developed a fuzzy linear programming model by comparing fuzzy numbers. In a separate study, G. Bortolon and R. Degani (1985) [11] provided an overview of ranking fuzzy numbers and discussed various ranking methods. These methods are highly beneficial for addressing fuzzy transportation problems. Similarly, S. H. Chen (1985) [16] outlined operations on fuzzy numbers based on principles. *Then* M. S. Chen (1985) [15] solved an assignment problem of Fuzzy numbers. In 1988, J. J. Buckly utilized fuzzy numbers to address possibilistic linear programming problems involving triangular fuzzy numbers, ultimately delivering precise solutions for the problems at hand. Gani et al (2006) [26] introduced transportation problems in terms of fuzzy numbers, known as two sta Allahaviranloo et al (2008) [5] have presented a new approach for handling fully fuzzy linear programming problems (FFLP). They utilize a linear ranking function to de-fuzzify the FFLP and demonstrate the equivalence between two issues using theorems. Effati et al (2010) [22] have concentrated on linear programming problems with right-hand side coefficients in fuzzy numbers. They specifically consider the case of fuzzy numbers with linear membership functions. Additionally, they have proposed the "modified subgradient method" and "method of feasible directions" to address these problems. Myo (2012) [42] has investigated two different graphical methods for real-life problems and has introduced a novel program for linear programming problems, which is based on Microsoft Visual Basic Programming Software. This software is highly beneficial for obtaining fast and easy optimal solutions for linear programming problems. Xiaobo et al (2013) [38] have reviewed the latest developments in the distributional study of mixed integer linear programs with random objective coefficients. They have also explained the complexity of these models, conic programming formulations, and the use of persistency models. The main message they aim to convey is the importance of these models and methods in addressing linear programming problems.

Li et al. (2014) [39] analyzed the characteristic features and deficiencies of current stochastic programming methods, highlighting the issue of higher computation complexity. They introduced the concept of reliability coefficient and a quasi-linear processing sample based on expectation and variance. Furthermore, they explored the relationship between the reliability coefficient and reliability degree, as well as the selection method for the reliability coefficient. Subsequently, they developed a quasi-linear programming model for stochastic transportation problems and evaluated its performance through a case study. The findings suggest that this model offers good interpretability and operability, effectively addressing transportation issues in complex stochastic environments or with incomplete data. On the other hand, Kumar et al. (2013) noted that the existing general form of perfectly fuzzy linear programming problems, where all parameters are represented by flat fuzzy numbers, is only valid in the absence of negative signs. Nevertheless, the current standard format of the perfectly fuzzy linear programming problem becomes invalid in the presence of a negative sign. A new standard format is proposed as a result. Rajarajeshwari et al (2014) [49] have introduced an innovative approach that utilizes ranking methods to tackle the Fully

Fuzzy Linear Programming Problem (FFLP). This method involves converting the given FFLPP into a Crisp Linear Programming (CLP) Problem with constraints on bound variables and utilizing Robust's ranking technique to find a solution. Additionally, the optimal solution obtained through the proposed method is compared with that of the existing method. Saati et al (2015) [52] have introduced a new method for solving the Fuzzy LP (FLP) problem, where fuzzy numbers are used to represent the right-hand side parameters and decision variables. A new fuzzy ranking model and additional variable are incorporated in the proposed FLP method to obtain both fuzzy and crisp optimal solutions by solving a single LP model. Moreover, an alternative model with deterministic variables and parameters derived from the proposed FLP model is also introduced. Notably, the result of the alternative model is equivalent to the crisp solution of the proposed FLP model. Ghadle and Muley (2015) [27] have proposed a viable solution for the real assignment problem of Laptop selection using MATLAB coding. Barrios et al (2016) [8] have conducted a study on a specific type of piecewise linear system. Their hypothesis suggests that the semi-smooth Newton method, when applied to this system, linearly converges to a solution under a mild assumption. The authors have also outlined how the sequence generated remains bounded regardless of the starting point, along with providing a formula for any accumulation point. In their research, they have explored the application of this method to solve convex quadratic programming problems with positive constraints. The numerical results indicate that Newton's semi-smooth method is effective in accurately solving large-scale problems in a few iterations. Rao and Srinivas (2016) [50] have developed an efficient algorithm to obtain the optimal solution for an Assignment Problem, aiming to minimize computational costs. Yang and colleagues (2016) [65] presented a lattice linear programming problem involving maximum product fuzzy relational inequalities in the context of optimizing the management model for wireless communication radiation base stations. The solutions for maximal product fuzzy relation inequalities were analyzed by comparing them to those of the corresponding maximal product fuzzy relational equations. A solution matrix method was developed to address the proposed problem without the need to identify all (quasi-)minimal solutions of the constraints. Chretien and Corset (2016)[17] investigated the probability of the reviewed time in intricate aging systems, specifically focusing on the shortest path distance in a Directed Acyclic Graph with varying costs on edges.

Zhu and colleagues (2016) [70] introduced a new linear programming formulation called ν-LPNPSVM. This formulation, ν-NPSVM, has been proven to outperform twin support vector machines (TWSVMs) and is parameterized by the value of ν to effectively manage the number of support vectors. By adding a 1-norm regularization term to ν-LPNPSVM, it transforms from a quadratic programming problem to a linear programming problem, making it easier to interpret. Additionally, they incorporated the kernel function directly into the construction for the nonlinear case. Ghadle and Muley (2015) [27] have proposed a viable solution for the real-world assignment problem of laptop selection using MATLAB coding. Barrios et al. (2016) [8] have conducted a study on a specific type of piecewise linear system. Their hypothesis suggests that the semi-smooth Newton method, when applied to this system, converges linearly to a solution under certain conditions. The authors have also outlined how the generated sequence remains bounded from any starting point, along with a formula for any accumulation point. In their research, they have explored the application of this method to solve convex quadratic programming problems with positive constraints. The numerical results indicate that Newton's semi-smooth method is effective in accurately solving large-scale problems in a few iterations. Rao and Srinivas (2016) [50] have developed an efficient algorithm to obtain the optimal solution for an assignment problem, aiming to minimize computational costs. Akpan et al. (2016) [4] utilized the simplex algorithm to address the issue of maximizing profit in linear programming by allocating ingredients to various bakery products. Their analysis revealed that producing 962 small bread units, 38 large bread units, and no giant bread units would result in optimum profits. Furthermore, it was found that large bread units significantly contributed to profit following small bread units, suggesting that focusing on producing and selling more small and large loaves would be beneficial in profit maximization. On the other hand, Das (2017) [19] introduced a modified algorithm for solving fully fuzzy linear programming problems to achieve fuzzy optimal solutions under equality constraints. Also, Ezzati et al. (Applied Mathematical Modelling, 39 (2015) 3183-3193) put forward a new algorithm to address fully fuzzy linear programming problems. Fokaidas and colleagues (2017) introduced a novel method for optimizing the design of building shells through solar radiation optimization. They utilized a nonlinear programming approach (simplex algorithm) to identify the most efficient building shape, focusing on functions associated with optimizing solar heat gain. The research focused on a one-story, convex, square, single-family house with two parallel sides. The calculations were conducted for two typical weather years (TMY) in two European cities, each representing a densely populated latitude in continental Europe with distinct summer and winter weather patterns. The findings of this study are anticipated to enhance the development of parametric design processes aimed at maximizing annual and seasonal sunlight exposure in buildings based on specific design criteria. Afroz (2017)[02] presented a novel method for addressing assignment problems using an algorithm and step-by-step solution. Through a comparison with two existing methods, the author determined that the proposed approach was easier to comprehend and offered a significantly improved solution in comparison to the current methods.

Woubante (2017) [63] examines a clothing business unit in Ethiopia as a case study. Data on monthly resources, product range, assets used per unit, and earnings per unit for each product were gathered from the organization. The collected statistics were utilized to determine the parameters of the linear programming model. The model was solved using LINGO 16.0 software. The study's results indicate that the company's revenue could increase by 59.84%, meaning that the total monthly earnings of Birr 465,456 could be raised to Birr 777,877.3 per month by implementing linear programming models to fulfill customer orders. The company's revenue could also increase by 7.22% if the linear programming formula does not need to account for customer orders. Capitanescu et al (2017) [13] presents a novel local-based iterative LS method that relies on first-derivative approximation and linear programming (LP),

The objective is to direct the exploration in different directions within the target space. In addition, this research employs the Directed Local Search (DS) method for constrained MOO problems. These approaches are utilized for bio-target optimization, which involves comparing costs and life cycle assessments based on environmental impacts in drinking water treatment facilities. The results obtained demonstrate that the proposed method is a promising local search technique that outperforms the directed search method. Rai et al (2017)[47] introduce a new method to obtain the optimal solution for Assignment problems namely an alternate method for solving assignment problems. Ying and colleagues (2018)[66] researched multi-player multi-objective game models with linear objectives. They introduced two methods for solving the model based on duality theory and Karush-Kuhn-Tucker (KKT) conditions. The duality-based approach demonstrates that solving the Pareto equilibrium of the primal problem is analogous to solving a multi-linear system. On the other hand, the KKT-based approach illustrates that achieving Pareto equilibrium can be done by solving a linear complementarity problem. In their study, they applied these methods to analyze a supply chain competition problem utilizing the game model. Rodias and colleagues (2018)[51] utilized a blend of simulation and linear programming to replicate a logistics system for organic fertilizer (in the form of slurry). The approach is tailored to specific crops and fields while factoring in unique agricultural, legal, and other limitations, all aimed at minimizing optimal annual expenses. Tillage and seeding tasks were integrated due to their direct impact on the organic fertilizer distribution. In the initial scenario, the optimal costs for both crops were assessed across a total area of 120 hectares. Three adjusted scenarios were presented. The first scenario involved an additional tractor, resulting in a 3.8% decrease in total annual costs compared to the baseline scenario. In the second and third adjusted scenarios, fields with high nitrogen requirements were taken into consideration alongside the operation of 1 or 2 tractors, leading to savings of 2.5% and 6.1%, respectively, in comparison to the baseline scenario. Ultimately, it was determined that the distance between manure production and the field could potentially lower costs by 6.5%. Faudzi and colleagues (2018)[24] classified assignment issues into two primary categories: Time Table problems and assignment problems. Timetable issues involve exam, course, and school timetables, while assignment problems pertain to student project allocations, new student assignments, and room assignments. Moreover, the research discusses the challenging and flexible boundary conditions implicated in these problems. It also introduces diverse approaches to tackle various attribution issues, offering guidance and feasible problem-solving directions based on contemporary practices. This comprehensive overview consolidates an in-depth analysis of attribution challenges in education, fostering a deeper comprehension of different attribution problems and proposing various solution strategies.

Aboelmagd (2018)[1] discusses linear programming concepts that emphasize the importance of time cost and timing issues in project management. The study utilizes LINGO software to formulate linear programming models aimed at solving cost and time-related problems. However, the model developed has certain limitations when applied to the project under investigation. Despite this, site managers can still utilize it to forecast the consequences of construction decisions and explore different strategies to achieve management objectives. On the other hand, Voskaglou (2018)[60] proposes a novel approach to address these issues by ranking fuzzy numbers and solving the resulting linear programming problem based on standard theory. The decision variables' values are then converted into fuzzy numbers to present an optimal solution in a fuzzy format, although this process must be carefully validated to ensure accurate representations.

Muruganandam and Hema (2019)[40] presented a genetic algorithm approach to obtain the fuzzy optimal solution for a fuzzy assignment problem by taking all parameters as triangular fuzzy numbers. Solaja et al (2019)[55] conducted a study where they applied linear programming techniques to solve manufacturing planning issues in a feed mill organization. The Linear Programming model was created based on data obtained from the organization and processed using Management Scientist Version 5.0. The research findings demonstrate that profits were improved by reorganizing the product range and discontinuing less productive products. By following this recommendation, the company can implement the results of the linear programming techniques in production planning to enhance monthly profits. Oladejo et al (2019) [43] This paper deals with the utilization of the optimization principle in optimizing revenues of a production industry by using linear programming to observe the manufacturing cost to determine its profit. This paper makes use of secondary information accumulated from the record of the Landmark University Bakery on five categories of bread production in the firm which include Family loaf, sliced family bread, Chocolate loaf, medium-sized bread, and small-size bread. A problem of this nature was identified as a linear programming problem, formulated in Mathematical terms, and solved using AMPL software.

Sreeja (2019) [57] introduced the method to carry effectiveness to solve assignment problems with a new technique formulated. Ghanbari and colleagues (2019) [28] explore various types of fuzzy linear programming problems based on models and solution methods. Initially, they analyze fuzzy linear programming problems involving fuzzy decision variables and fuzzy parameters, along with the associated duality results. Subsequently, they investigate fully fuzzy linear programming problems where all variables can be fuzzy. The majority of methods utilized to address such problems are rooted in ranking functions, α-cuts, the use of duality results, or penalty functions. Through these approaches, the authors address crisp formulations of the fuzzy problems. More recently, heuristic algorithms have also been introduced. In these methods, some authors directly solve the fuzzy problem, while others approximate the crisp problem.

Rai and Khan (2019) [46] provide an overview of the assignment problem, which entails efficiently matching elements from multiple sets. The complexity of the problem is determined by the number of sets involved. The objective is to recognize these complexities and assist researchers in creating customized versions of the assignment problem for particular purposes. Anne et al (2020) [7] conducted a study on the application of linear programming in decision-making for firms, utilizing a hypothetical example. The research aimed to implement a linear programming model to minimize the production time of hair and body cream at Seun Cream Factory Limited. By utilizing the simplex method, the results indicated that to maximize profit, 6.5 units of hair cream and 27.7 units of body cream should be produced, leading to a total profit of ₦1107.75. The study concluded that both types of creams should be manufactured to meet customer demand but to optimize profit, more body cream production is recommended, as it contributes significantly to the company's profitability. Furthermore, the study stressed the importance of adopting optimization techniques to improve the efficiency and effectiveness of a firm's operations, irrespective of its scale. Solaja et al (2020) [56] addressed course allocation issues in Nigerian tertiary institutions by utilizing an assignment model to enhance lecturers' effectiveness with students. They designed a well-structured questionnaire to gather data from students, which was then analyzed using the Hungarian method. The research findings indicated that the adoption of the assignment model could increase lecturers' effectiveness in decision-making by 13.20% for each topic. This study contributes to enhancing students' comprehension of different subjects. The results suggest that assignment models are an efficient way to tackle course allocation challenges in tertiary institutions. Institutions are encouraged to implement the assignment model for maximum advantage and to elevate the quality of education in the country. San and E Murugesak Kiammal (2020) [41] present a novel approach to Assignment Problems (AP) called TERM, which aims to address a wide range of APs with minimal mathematical computations. The TERM method works by transforming a given cost matrix into a matrix of opportunity costs (MOC) that contains at least one zero in each row and column and then assigning values to these zero entries. This method provides the best solution for the specific AP at hand. To validate the effectiveness of the TERM method, 30 classical benchmark instances from existing literature were examined. The simulation results demonstrate that the proposed TERM method is highly efficient, delivering optimal solutions comparable to HAM in 24 instances (equivalent to 80 seconds). Indra and Jayalakshmi (2020) presented novel techniques for the row-column minima method introduction to determine an optimal interval solution for the full interval integer transportation problem (FIITP). The FIITP is split into lower (LBITP) and upper (UBITP) transportation problems, and the optimal interval solution is derived by applying the row-column minima method. Biswas and Sumaya (2020) [10] examined the impact of the COVID-19 pandemic on the healthcare system, particularly the need for dedicated COVID-19 units in hospitals due to an increase in the number of patients requiring intensive care. One of the major challenges in healthcare has been optimizing the number of nurses in each shift to meet the seasonal peak in demand for care, especially as there is a shortage of nurses willing to work in COVID-19 units due to the risks involved. As a result, there is a growing need for an optimized nurse schedule that can be implemented efficiently while meeting client demand and other requirements such as flexibility in the workplace. This study presents a linear programming problem aimed at improving the nurse scheduling process and maximizing the fairness of the resulting schedule. A mathematical model is formulated and solved using the big M simplex algorithm to minimize the required number of nurses, along with linear optimization of the scheduling process. Data from four different hospitals were collected to ensure statistical consistency, and a case study from a major hospital was used to demonstrate the method and determine the optimal solution. As a result, an optimized schedule has been proposed that minimizes the required number of nurses by balancing the trade-off between increasing demand and reducing the workforce.

Jung and Kim (2020) [33] introduced an algorithm aimed at identifying the smallest set of axes capable of representing building geometries and city layouts in two dimensions. While axis lines are valuable for spatial organization analysis in spatial syntax, current methods for selecting axis lines often fall short in terms of optimizing the solution. The algorithm proposed in the study utilizes linear programming to derive a minimal set of axis lines. To reduce the number of axes needed to represent the overall geometry of buildings and city plans, a linear programming problem is formulated where a set of axes represents the entire geometry. Each axis line must intersect with every extension of the wall edge at the angle of overlap. If a solution to this linear programming problem exists, it is guaranteed to be the most optimal. Nevertheless, the solution to this linear programming problem may include isolated lines, which are not ideal for axial line analysis. To prevent isolated axes, the paper introduces a new formulation by incorporating additional constraints to the original one. Through an examination of a modified linear programming problem using various 2D architectural plans and spatial layouts, the study demonstrates that the proposed algorithm can ensure a minimal set of axes representing the 2D geometry. This modified linear programming problem effectively eliminates orphaned axes during the axis reduction process.

Ammar and Emsimir (2020)[6] introduced an algorithm aimed at addressing fully rough integer linear programming (FRILP) problems to identify rough value optimal solutions and decision rough integer variables. In this approach, all parameters and decision variables in both the constraints and the objective function are considered as rough intervals (RIs). The algorithm proposed enables the search for the optimal solution within the broadest range of potential solutions. A flowchart is included to illustrate the steps involved in problem-solving. On the other hand, Rajarajeswari and Maheswari (2020)[48] tackled a transportation problem with mixed constraints, where all parameters are treated as integer intervals. They also managed to solve the fully integer interval transportation problem without converting it into a crisp transportation problem. A numerical example was presented to validate the argument, and the results were compared with existing methods. Thipwiwatpotjana et al. (2020)[59] put forward six different types of solutions (weak, strong, tolerance, control, left-localized, and right-localized solutions) for systems of equations, along with four different types of solutions (weak, strong, tolerance, and control solutions) for systems of inequalities. They also established the necessary and sufficient conditions for verifying their solvability based on systems of linear inequalities that depend on the sign of variables. Consequently, an interval linear programming problem with nonnegative variables and two-sided interval linear constraints can be addressed using a standard linear programming approach.

Yang et al (2020)[65] proposed a method for solving fuzzy multi-objective linear fractional programming (FMOLFP) problems using a technique based on superiority and inferiority measures (SIMM). In this model, each fuzzy desire related to fuzzy goals and some constraints are described using fuzzy numbers. SIMM is used to handle fuzzy numbers in constraints for the maximum membership value, while a linear goal programming technique is applied to address the challenge where the fractional objectives are fuzzy goals. The algorithm is demonstrated by solving an optimization problem related to agricultural planting systems. The results indicate that the optimal acreages for winter wheat and summer corn are 38,386.4 ha and for cotton, it is 20,669.6 ha. Due to the high risk in cotton cultivation, the ratio of grain planted area to cotton planted area is found to be unreasonable. Thus, the government must provide improved support to enhance the farmers' enthusiasm for cotton cultivation and sustain the long-term development of the cotton market.

Kane and colleagues (2021)[34] introduced a method for solving primal and dual linear programming problems using trapezoidal fuzzy numbers (LPTra). This method involves transforming the problems into two linear programming problems with a specific number of intervals (LPIn) and incorporating new arithmetic operations between interval numbers and fuzzy numbers. The optimal solutions obtained from this approach adhere to both the Strong Duality Theorem and the Complementary Looseness Theorem. To provide a clearer understanding, numerical examples are utilized to showcase the effectiveness and practicality of the proposed technique. In his 2021 study, Wieloch introduces a novel algorithm merging binary programming with scenario planning, incorporating an optimism factor reflecting the manager's risk attitude. This method is tailored for single-shot decisions, where the selected alternative is implemented only once, and restricts the use of mixed strategies involving multiple decision variants. Ghoushchi et al (2021)[29] proposed a novel technique for addressing fully fuzzy linear programming (FFLP) problems. Likewise, in this research, the authors investigate a fresh method for solving the FFLP problem by incorporating fuzzy decision parameters and variables utilizing triangular fuzzy numbers. They also suggest a tactic based on alpha-cutting theory and modified triangular fuzzy numbers to achieve the optimal complete fuzzy solution for real-world problems. This approach treats the problem as a complete fuzzy problem and resolves it by utilizing the newly defined triangular fuzzy numbers to optimize the decision variables and the objective function.

Wang (2022)[61] conducted a detailed study on the mathematical model of fuzzy linear programming, focusing on the expressions of the f(ti) and g(t0) functions under elastic constraints and their optimal solutions. The application of fuzzy linear programming in agricultural economic management in five urban areas of Zhengzhou was then explored to assess its effectiveness. The findings revealed that the optimal solutions for the f(ti) and g(t0) functions varied with different degrees of elasticity, each having a unique optimal solution. Furthermore, the mathematical model based on fuzzy linear programming demonstrated its ability to accurately predict the pull force in agricultural economic management, offering valuable theoretical support for its practical application. Sharma and Tewatiya (2022)[54] introduced a linear programming problem formulation using a real-world scenario, emphasizing the significance of experienced teachers, efficient management, and student population in the growth of educational institutions. The application of linear programming in higher education institutions aims to maximize the university's profit by determining the optimal number of students required for the solution.

Ekanayake and colleagues (2022)[23] present a comprehensive summary of different transportation-related issues and mathematical models. This can be utilized to effectively address various commercial challenges associated with product distribution, often known as transportation problems. The study by Drodevic et al (2022)[20] demonstrates the integration of linear programming and multi-criteria decision-making (MCDM) models. Initially, linear programming was utilized to optimize production, leading to multiple potential solutions along line segment AB. A set of criteria was established and assessed using a modified fuzzy stepwise weighted ratio analysis (IMF SWARA). To arrive at the ultimate solution, a novel rough con-pro ranking of alternatives from distance to ideal solution (R-CRADIS) was devised and verified through comparative analysis. The findings indicate that combining linear programming with fuzzy rough MCDM models can offer an effective approach to addressing specific optimization challenges.

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