A cellular automaton model of large-sized railway passenger station based on lagrange coordinate system

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Abstract: With the rapid development of high-speed rail and passenger dedicated lines, the load in the throat area of railway passenger stations is increasing. This paper uses the multi-dimensional cellular automaton model and the NS model as the basis for the Lagrange coordinate system. The train is considered as a one-dimensional continuous medium model. In the fluid, a cellular automaton model for train operation in the throat area of a railway passenger terminal is proposed. The results show that this model can depict the running status of the train in the throat area realistically, and can provide a theoretical basis for subsequent research and analysis.

Keywords: lagrange coordinate system; railway passenger station; throat area; cellular automaton.

1. Introduction

Large-sized railway passenger stations are the main places for train departure, arrival, passing and passenger transportation. With the rapid growth of passenger volume, improving passenger station capacity and service level is one of the main purposes of railway network development. In recent years, with the marketization of railway transportation and the rapid development of high-speed railways, scholars did a lot of researches on large-sized railway passenger stations. Since large-sized railway passenger stations are the nodes and hubs of the railway network, the operating efficiency and passing capacity of the passenger stations directly affect the transportation capacity of the entire road network system [1].In the existing researches, the train operation plan and timetable are mostly based on the lines [2], mainly focus on the passing capacity of single line [3], and do not involve the specific operation of busy passenger station. However, the passenger station with complex turnout group and large number of tracks in throat area is exactly the station where passenger trains operate most frequently. The operation route in throat area of large-sized railway passenger station is heavy, and the number of departure, final arrival, passing and stopping trains is large [5]. Although the passenger train operation is relatively fixed, the arrangement of shunting operation route and the application of arrival departure track are very flexible [6].

Therefore, the throat area of the railway passenger station has received increasing attention from scholars. Specifically, Cai studied the main equipment and technological process at large -scale railway passenger station, and found that the turnout group, the arrival and departure line, shunting operation, vehicle taking and delivery, the kind of operation are five important factors affecting the throat carrying capacity of large-scale passenger station[7]. Ding emphasized the importance of determining the size of arrival and departure lines of large-scale railway passenger stations, analyzed the main differences between passenger stations on existing lines and large-scale railway passenger stations, and pointed out the factors that need to be considered when determining the number of arrival and departure lines between the two, and gives the main factors affecting the determination of the scale of arrival departure tracks of large-scale railway passenger stations[8]. Li analyzed the influence of route arrangement in throat area of passenger station on the allocation of arrival and departure lines, and summarized two constraint conditions for application optimization of the arrival and departure lines: the basic constraints of the arrival and departure line allocation and the train operation diagram constraints, and proposed an optimization model of arrival and departure line utilization based on balanced utilization of equipment[9].

Most of the research on rail transit adopts the classical mathematical modeling method. However, due to its complicated calculation process and difficulty in solving, scholars gradually begin to use cellular automata to simulate it. By improving the NS model, Ning proposed a cellular automata model under three display fixed block and moving block modes, and found the characteristics of rail traffic flow such as the speed effect of front car in train tracking operation, which enriched the research on cellular automata model of rail transit [10]. Wang proposed a single track railway parallel operation primitive cellular automata model based on the number of tracks, station spacing and basic operation time, and analyzed the relationship between these factors and section passing capacity, making up for the shortcomings of existing calculation methods of passing capacity [11]. Taking into account the characteristics of short driving intervals and large interactions between trains of urban rail transit, Cai presented a moving block rail transit cellular automatamodel, and verified the effectiveness of the model [12]. Based on the "high and medium speed mixed" transportation organization mode, Han proposed a cellular automata model for train tracking under moving block conditions, and analyzed the influence of buffer time, proportion of medium speed trains and station spacing on high-speed railway carrying capacity and average train speed [13].

To sum up, although many scholars have studied the passing capacity and rail transit flow characteristics of passenger station throat area, the simulation of throat area in large railway passenger station with CA model is almost blank. Therefore, in this paper, combined with the one-dimensional cellular automata model, we improved the cellular attributes and its evolution rules based on Lagrange coordinates, and transformed the train operation rules in throat area into a one-dimensional cellular automata model to solve the problems of turnout conflict and arrival departure track occupation in train operation rules, and simulated the train operation in throat area.

2. Lagrange coordinate method

2.1. Fundamental concepts

Lagrange coordinate method is based on the particle in the flow field to study the fluid. The flow field is composed of many fluid particles which are continuously distributed. The research method of Lagrange form focuses on the change of physical quantities (velocity, density, flow, acceleration, etc.) on a single particle with time. This method can accurately describe the motion interface of a single particle, and can track the trajectory of a particle. Finally, the motion law of the whole flow field can be obtained by integrating the motion trajectories of all the particles [14].

In the research, the coordinates (x, y, z) of the particle position in the flow field are often used to distinguish different particles when $t = t_0$. In a specific space coordinate, a fluid particle only corresponds to a constant coordinate (x, y, z) . The physical properties (B) of any particle in the flow field, such as pressure, density, velocity, temperature, etc., can be expressed as follows:

$$B = B(x, y, z, t) \tag{1}$$

where (x, y, z) is called Lagrange variable or co-rotational coordinate, that is, the Lagrange coordinate.

The Lagrange coordinates of a spatial point can usually be defined by its initial position. Using the Lagrange method to explore the motion of fluid, particles can be tracked and distinguished by the invariance of Lagrange coordinates in a specific time. Traffic flow is a one-dimensional fluid. For this fluid, the Lagrange variable of a particle in a certain space can be regarded as the total mass. When this spatial point is transferred to another spatial point over time, the total mass remains unchanged. In this way, it is easy to establish equations of various parameters (variables such as mass, flow, acceleration, etc.) and mass conservation equations. Applying this method to traffic flow helps to establish the link between the macro and micro models of traffic flow.

2.2. The difference between Lagrange coordinate and Euler coordinate

Euler coordinates refer to space coordinates. For particle motion, it studies the state of different particles passing through a certain fixed point in space. Lagrange coordinates refer to the material coordinates. Lagrange coordinates refer to the material coordinates. For the movement of a particle, it is to follow the particle to study the motion state of the particle. In short, Euler coordinates are fixed in space, Lagrange coordinates are fixed in materials, so Euler coordinates are often called space coordinates, and Lagrange coordinates are often called material coordinates.

Assuming that there is a function $\phi(x,t) = at^2 + bx^2$, we can see that (x,t) is related to both spatial position x and time t. Taking the derivative in the above two coordinate systems respectively, then:

(1) In the Lagrange coordinate, since the coordinates are fixed to the material points, the corresponding positions are also different at different times, so:

$$\left(\frac{D\phi}{Dt}\right)_L = \frac{\phi_2 - \phi_1}{\Delta t} = \frac{\left(at_2^2 + bx_2^2\right) - \left(at_1^2 + bx_1^2\right)}{\Delta t} = a(t_1 + t_2) + \frac{b(x_2 - x_1)(x_2 + x_1)}{\Delta t} \qquad (2)$$
$$= a(t_1 + t_2) + ub(x_2 + x_1)$$

where, u represents the speed of movement during time Δt, and it is also a transport process.

(2) In the Euler coordinate system, since it is a spatial coordinate, even if Δt time has passed, its reflection in the function (x,t) is only the effect of time change, then:

$$\left(\frac{\partial \phi}{\partial t}\right)_E = \frac{\phi(t_2)-\phi(t_1)}{\Delta t} = \frac{\left(at_2^2+bx_2^2\right)-\left(at_1^2+bx_1^2\right)}{\Delta t} = a(t_1+t_2) \tag{3}$$

Comparing the results with Lagrange coordinates, we can see that the result under Euler coordinate is a kind of partial derivative in a sense. From the comparison of the above two results, a commonly used operator can be derived:

$$(\frac{D}{Dt})_L = (\frac{\partial}{\partial t})_E + u\nabla \tag{4}$$

2.3. The relationship between Lagrange coordinate and Euler coordinate

Lagrange coordinate method and Euler coordinate method are essentially the same. If the fluid particle marked as parameter (x, y, z) moves to the point (a, b, c) in space at the time t, then there is:

$$B = B(a,b,c,t) = B[a(x,y,z,t), b(x,y,z,t), c(x,y,z,t), t] = B(x,y,z,t) \tag{5}$$

Therefore, the two methods express the same movement, and they are interrelated and can be converted to each other.

2.4. Basic relations of Lagrange traffic flow

Assuming that $M(x,t)$ represents the total mass of the fluid in the upstream section at position x at time t, the density of the fluid can be expressed as follows:

$$\rho(x,t) = \lim_{\Delta x \to 0} \frac{M(x+\Delta x, t) - M(x,t)}{\Delta x} = M_x(x,t) \tag{6}$$

and the flow rate can be defined as follows:

$$q(x,t) = -\lim_{\Delta t \to 0} \frac{M(x, t+\Delta t) - M(x,t)}{\Delta t} = -M(x,t) \tag{7}$$

In the traffic flow theory, M is the number of vehicles at position x at time t .according to $M_{xt} = M_{tx}$, combining equations (6) and (7), the mass conservation equation can be obtained:

$$\rho_t + q_x = 0 \tag{8}$$

Under the assumption of a first-order continuum, the total number of trains upstream at a given spatial point remains constant during the movement in the road section over time. In addition, without considering the vacuum condition, when t is constant, according to equation (6), M is strictly increasing with respect to x . Therefore, under the Lagrange system established according to equations (6) and (7), replacing x with M, it is easier to observe the train moving in the section at any time [15]. Equation (8) is equivalent to:

$$s(M,t) = x_M(M,t) \tag{9}$$

Where $s(M,t) = (\rho(x,t))^{-1}$ is the specific volume of (x,t) at (M,t) , and $x(M,t)$ represents the position of the mass point M at time t. Similarly, the velocity of the mass point M at time t can be defined as follows:

$$v(M,t) = \lim_{\Delta t \to 0} \frac{x(M,t+\Delta t) - x(M,t)}{\Delta t} = x_t(M,t) \tag{10}$$

Furthermore, the velocity $v(x,t)$ of the fluid at the position x and at time t can also be replaced with the velocity $v(M,t)$ of the particle, namely

$$v(M,t) = v(x,t) \tag{11}$$

If the solution is in a smooth region, according to $x_{Mt} = x_{tM}$, the partial derivative of the implicit function $x = x(M,t)$ can be obtained,

$$x_t(M,t) = -\frac{M_t(x,t)}{M_x(x,t)} = \frac{q(x,t)}{\rho(x,t)} \tag{12}$$

Combining equation (10), we can get:

$$q(x,t) = \rho(x,t)v(x,t) \tag{13}$$

In the one-dimensional continuum model, the physical parameters of the fluid are only related to the changes in the spatial point of the fluid, that is, the position coordinates. When the physical parameters of the fluid cross section are evenly distributed, or the average value of the physical parameters is calculated according to the fluid cross section, it is one-dimensional flow. Therefore, the position update of the fluid along the direction of movement is a one-dimensional flow, thereby simplifying the evolution rules of the model.

3. Cellular automata model of throat area in large-sized railway passenger station

Large-sized railway passenger stations are the nodes and hubs of the railway network, and are often the stations with the most frequent train operations. Their operating efficiency and passing capacity directly affect the passenger transportation capacity of the entire road network system. In order to meet the technical operation requirements of trains in the large-sized railway passenger station, the layout of the throat area at both ends of the station is often extremely complicated, with complex turnout groups and a large number of strands. Therefore, it is important to simulate train dispatching operations in the throat area under the conditions of ensuring the safety of transportation operations and achieving high-speed comfort to improve the punctuality of receiving and dispatching trains. In this paper, a multi-one-dimensional cellular automata model is proposed in the Lagrange coordinate system to simulate train operation in the throat area of a large railway passenger station.

3.1. Basic settings

The throat area is composed of turnout groups, arrival and departure lines, main lines and various line terminal connections, crossovers, ladder lines and other parts. Each part has its own particularity, so that the train has different operating rules in the actual operation process. Therefore, it is necessary to distinguish the operation process of each part in the simulation, otherwise it is difficult to truly simulate the operation process of the train in the throat area. In addition, the train's running path in the model is fixed, but in order to shorten its line occupation time and improve operation efficiency, it is usually set to the route with the shortest occupation time.

Generally, the line terminal connecting line, crossover, ladder line are collectively referred to as connecting lines, and the rest are ordinary lines. For ordinary lines, its one-way section is regarded as a one-dimensional matrix. If a restricted section is involved when the train is running, it will decelerate by a certain amount and then run normally. At the connecting line, it is not only necessary to connect the train on the line, but also to let the train out on the next connecting section after a certain period of time. The model involves a complex two-dimensional matrix. It is difficult to simulate the throat area of large railway passenger station. Therefore, we simplify the model.

In this paper, combining the characteristics of the throat area, a multi-one-dimensional cellular automata model under Lagrange coordinates is proposed on the basis of the NS model. In the model, the main line and the arrival and departure lines are regarded as one-dimensional CA models, different lines are connected by connecting lines, and lane-changing rules are added to the connecting lines to realize the lane-changing behavior of trains at turnouts. Here, the connection lines are regarded as the connection cells to judge the logical relationship between the connection lines and then determine the time occupation relationship between the connection cells.

3.2. The rules

For easy understand, we give a schematic diagram of throat area of passenger station, as shown in Figure 1 below, in which the red circle represents the turnout and the red section represents the connecting line.

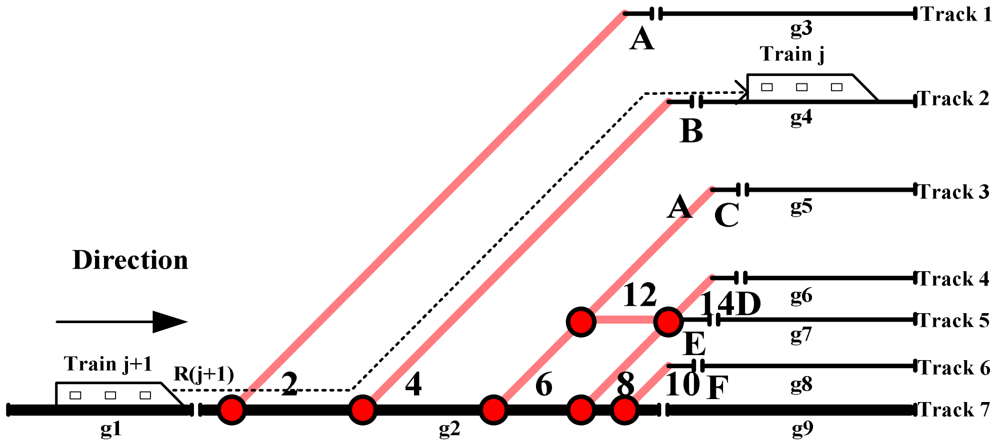

Figure 1. Schematic diagram of throat area of passenger station.

3.2.1. Time interval rule

Assuming that the train runs to the turnout, for the stop train, when the two trains j and j+1 occupy the same track $k \in K$ for operation, only when the tail of the preceding train j leaves the track and releases the corresponding track circuit section, the following train j+1 can enter the station. The arrival time t+1 of the following train j+1 represents the time after the train completes the receiving operation using this track, so the minimum operation interval time should be met between t+1 and the departure time d_t of the preceding train j.

$$t+1-d_t > \delta[j,j+1], j, j+1 \in T \tag{14}$$

where, δ is the sum of the time from the departure of the preceding train j to the release of the track k at its tail and the receiving operation time of the following train j+1. It is not only related to the parameters of the two trains, but also related to their incoming route.

3.2.2 Route conflict rules

As shown in the schematic diagram of the throat area of the passenger station in Figure 1, there are 7 tracks, 7 turnouts and 9 track circuit sections. Taking the section releasing mode as an example, here a route is represented by the queue of track circuit section, and the arriving route of train j + 1 to the track 2 is $R(j+1)$={g1,g2,g4}, which means that route $R(j+1)$ is composed of three track circuit sections g1, g2 and g4 in turn.

Assuming that the departure operation of train j is earlier than the pickup operation of j+1, only after j releasing the track circuit section g4, can the arriving route $R(j+1)$ be prepared for j+1, so g4 is the critical track circuit section of these two operations. Let $e[t_j, g_j]$ denotes the time when train j releases g4, and let $b[t_{j+1}, g_j]$ to indicate the time when train j+1 starts to occupy g4, then the route conflict rules of two trains can be expressed as follows:

$$b[t_{j+1}, g_j] - e[t_j, g_j] > 0 \tag{15}$$

This rule can be used to judge the operation conflict of trains in throat area, which can provide basis for route selection.

3.3. Line rules

In this paper, each section of the throat area is taken as a one-dimensional cell model, assuming that the length of line i is l_i. When the section is occupied, that is, there is a train running on the section, assuming that the cell length from the head of the train (j) to the end of the section in the direction of travel at time t is $s_{ij}(t)$, and the distance from the end of the train to the starting point is $x_{ij}(t)$, then $l_i = s_{ij}(t) + x_{ij}(t) + p_{ij}$. Assuming that the speed of the train is $v_{ij}(t)$, the distance between train j and its preceding vehicle j+1 satisfies $d_{ij}(t) = s_{ij}(t) - x_{i(j+1)}(t)$, as shown in Figure 2. For intuitive representation, each grid in the figure represents multiple cells, and train j and train $j+1$ have the same length.

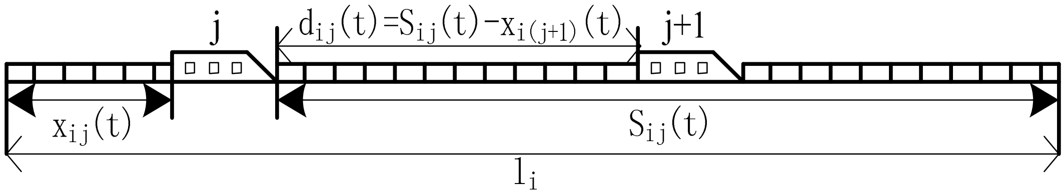

Figure 2. Schematic diagram of train operation rules in the throat section.

3.3.1. Ordinary line

In the one-dimensional model of the road section, the train operation rules are consistent with the single lane model. The train is represented by n cells, and the evolution of the cell state is used to realize the moving process of the train in the model. The following equation is satisfied when crossing the turnout is not considered:

$$s_{ij}(t) \neq 0 \,\&\&\, l_i - s_{ij}(t) \geq p_{ij} \;||\; x_{ij}(t) > p_{ij} \tag{16}$$

where, p_{ij} is the length of the train.

The train running on this section satisfies the four evolution rules of the NaSch model.

①If the distance between train j and its preceding train on section i satisfies the acceleration conditions of the train, namely,

$$d_{ij}(t) > \min(v_{ij}(t) + 1, v_{max}) + d \tag{17}$$

it will accelerate to reduce the time occupied by the line.

$$v_{ij}(t+1)=\min(v_{ij}(t)+1, v_{max}) \tag{18}$$

②If the distance between train j and its preceding train on section i or the remaining distance from the road section cannot meet the current speed, namely,

$$d_{ij}(t)<(v_{ij}(t)+d)\,||\,s_{ij}(t)<v_{ij}(t) \tag{19}$$

then,

$$v_{ij}(t+1)=(d_{ij}(t)-d)\,||\,v_{ij}(t+1)=s_{ij}(t) \tag{20}$$

it will decelerate to avoid rear end collision or exceeding the end of the section.

③Motion. The train updates its position at the adjusted speed.

$$s_{ij}(t+1)=s_{ij}(t+1)-1-v_{ij}(t+1)) \tag{21}$$

3.3.2. Arrival departure line

If the train is on the arrival departure track, it is necessary to judge whether the occupied time of the train on the arrival departure track meets the set stopping time of the train during the update. If not, then

$$line\_time(i,s\_l)=line\_time(i,s\_l)+1 \tag{22}$$

If the stop time is met, then

$$\begin{aligned}&if\ \ line\_time(g,s\_l)>=t\_stay\\&\quad line\_state(g,s\_l)=0\\&\quad line\_time(g,s\_l)=0\\&\quad new\_v=1\end{aligned} \tag{23}$$

Assign the status of the arrival departure track occupied by the train as 0, assign the stop time to 0 and the speed to 1, to make it leave the occupied arrival departure track.

3.4. Rules for the turnout and connecting line

Regarding the turnout and the connection line as the connection cell (the red section in Figure 1), and the occupation of turnout by train is regarded as occupation of time. When the occupancy time is reached, the turnout is released and the cell shows empty. The program design includes the following situations:

①The connection cell is empty. When the next route in the planned train operation routes is empty ($R(2-A)=0$), and the track circuit g2 of this section is unlocked, that is, the next section can make the train leave the turnout at the turnout, and the train will change the track under the condition of ensuring safe distance. Then the status of the route changes to occupied ($R(2-A)=1$), and the stop time is calculated cumulatively.

②The connection cell is occupied. The train will wait in place until the connection cell is free.

③When the accumulated time of the occupied route meets the set stopping time, the speed of the train is assigned as 1 to leave the arrival departure track, and the occupation status of the route is changed from occupied ($R(2-A)=1$) to empty ($R(2-A)=0$).

3.5. Simulation

On the basis of the above theory, combined with the actual data of the throat area of a passenger station, a simulation model of the throat area of the passenger station is constructed. The model is written in MATLAB and displayed graphically. The program operation interface is shown in Figure 3, where the grid at the turnout is the connected cell, and the black grids indicate that the place are occupied by trains. The simulation of each section is independent of each other, and the specific settings are as follows:

① The length of the train is fixed, occupying 20 cells.

②The model boundary is set as shown in Figure 3. The numbered position on the far right is the departure point of the model. The gray cell of track 7 is the entrance of the model. After the train stops at the station for the specified time, it will drive out of the model from the track.

③ In order to make the simulation display more intuitively, the throat area is scaled down in the simulation process, and every 5 cells on the road section are merged into a cell. When one cell is occupied, the combined cell is also occupied.

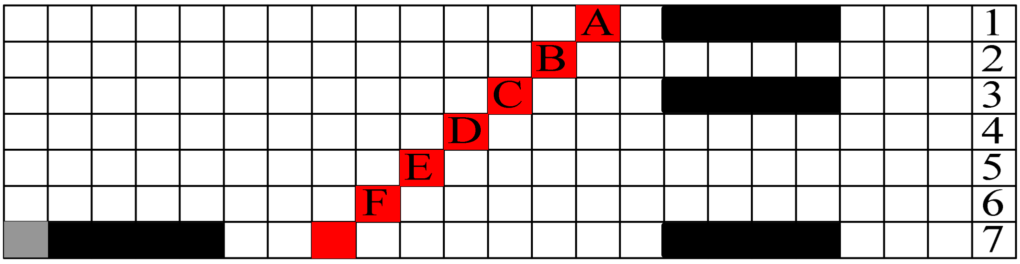

Figure 3. Schematic diagram of throat model simulation.

The simulation of the above model found that the train operation behavior is consistent with the actual situation, and there is no serious conflict or congestion during the busy time in the throat area, which achieves the expected effect of the model.

4. Conclusions

In this paper, the Lagrange coordinate system and cellular automata model are applied to the traffic simulation of passenger station throat area for the first time. After analyzing the difference between the train operation rules in the throat area of the passenger station and the main line, the cellular automata model of passenger station throat area in Lagrange coordinate system is proposed based on the one-dimensional cellular automata model and NS model. In the model, the train is regarded as the fluid in the one-dimensional continuum model. By updating the spatial point of the fluid, the train operation in the throat area of a passenger station is simulated. By adjusting the parameters to set the model rules for the throat area of the passenger station under different conditions and analyzing the corresponding simulation data, which can provide a theoretical basis for subsequent research and analysis.

Data Availability

All relevant data are within the paper and its supporting information on files.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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