Common fixed point theorems in 2-fuzzy metric spaces

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Abstract: The present paper deals with application of 'E.A. Like' property in proving common fixed point results in a 2-fuzzy metric space.

Keywords: 2-Fuzzy metric space, E.A. property, E.A.Like property, weakly compatible maps.

1.Introduction:

Zadeh [29] introduced the concept of fuzzy sets in 1965. Following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michalek [13] and George and Veeramani [5] modified the notion of fuzzy metric spaces with the help of continuous t-norm, which shows a new way for further development of analysis in such spaces. Consequently in due course of time some metric fixed points results were generalized to fuzzy metric spaces by various authors. Sessa [25] improved cmmutativity condition in fixed point theorem by introducing the notion of weakly commuting maps in metric space.Vasuki [27] proved fixed point theorems for R-weakly commuting mapping Pant [19] introduced the new concept of reciprocally continuous mappings and established some common fixed point theorems. The concept of compatible maps by [13] in fuzzy metric space is generalized by [9] by introducing concept of weakly compatible maps.Aamri and Moutawakil [1] generalized the notion of non compatible mapping in metric space by E.A. property. It was pointed out in [11], that property E.A. buys containment of ranges without any continuity requirements besides minimizes the commutativity conditions of the maps to the commutativity at their points of coincidence. More-over, E. A. property allows replacing the completeness requirement of the space with a more natural condition of closeness of the range. Some common fixed point theorems in probabilistic or fuzzy metric spaces by E.A. property under weak compatibility have been recently obtained in ([3], [13],[20], [28]). In this article, we defined a E.A. like property and proved common fixed point theorems of Sanjay kumar [18] removing some conditions by using E.A. like property.

Definition 1.1 [23] - A binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ is a continuous t-norms if $*$ satisfying conditions:

i. $*$ is commutative and associative;
ii. $*$ is continuous;
iii. $a*1=a$ for all $a\in[0,1]$;
iv. $a*b\le c*d$ whenever $a\le c$ and $b\le d$, and $a,b,c,d\in[0,1]$.

Examples: $a*b=$ min{a, b}, $a*b=$ a.b

Definition 1.2[5] - A 3-tuple $(X,M,*)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times(0,\infty)$ satisfying the following conditions , $\forall\ x,y,z\in X, s,t>0$,

$(f1)$ $M(x,y,t)>0$;
$(f2)$ $M(x,y,t)=1$ if and only if $x=y$.
$(f3)$ $M(x,y,t)=M(y,x,t)$;
$(f4)$ $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$
$(f5)$ $M(x,y,.):(0,\infty)\to(0,1]$ is continuous.

Then M is called a fuzzy metric on X. Then $M(x,y,t)$ denotes the degree of nearness between x and y with respect to t.

Example1.3 (Induced fuzzy metric [5]) – Let (X,d) be a metric space. Denote a $*$ b =ab for all $a,b\in[0,1]$ and let M_d be fuzzy sets on $X^2\times(0,\infty)$ defined as follows:

$$M_d(x,y,t)=\frac{t}{t+d(x,y)}$$

Then $(X,M_d,*)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric d as the standard intuitionistic fuzzy metric.

Definition 1.4: Two self mappings f and g of a fuzzy metric space $(X,M,*)$ are called compatible if $\lim_{n\to\infty}M(fgx_n,gfx_n,t)=1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty}fx_n=\lim_{n\to\infty}gx_n=x$ for some $x\in X$.

Lemma 1.5 : Let $(X,M,*)$ be fuzzy metric space. If there exists $q\in(0,1)$ such that $M(x,y,qt)\ge M(x,y,t)$ for all $x,y\in X$ and $t>0$,then $x=y$.

Definition 1.6 : Let X be a set, f and g selfmaps of X. A point $x\in X$ is called a coincidence point of f and g iff $fx=gx$. We shall call $w=fx=gx$ a point of coincidence of f and g.

Definition 1.7 [9] **:** A pair of maps S and T is called weakly compatible pair if they commute at coincidence points.

Definition1.8: Let f & g be two self-maps of a fuzzy metric space $(X, M, *)$ .we say that f & g satisfy the property E.A. if there exists a sequence $\{x_n\}$ such that,
$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$

Definition1.9:Let f & g be two self-maps of a fuzzy metric space $(X, M, *)$ .we say that f & g satisfy the property E.A.Like property if there exists a sequence $\{x_n\}$ such that,
$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in f(X)$ or $z \in g(X)$ i.e. $z \in f(X) \cup g(X)$

Definition1.10 (Common E.A. Property): Let $A, B, S, T : X \to X$ where X is a fuzzy metric space, then the pair $\{A, S\}$ & $\{B, T\}$ said to satisfy common E.A. property if there exist two sequences $\{x_n\}$ & $\{y_n\}$ in X s.t.

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = z \text{ For some } z \in X .$$

Definition1.11 (Common E.A. like Property) : Let A, B, S and T be self maps of a fuzzy metric space $(X, M, *)$, then the pairs (A, S) and (B, T) said to satisfy common E.A.Like property if there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = z$ where $z \in S(X) \cap T(X)$ or $z \in A(X) \cap B(X)$.

Example1.12:let $X = [0, 2)$ and $M(x, y, t) = \dfrac{t}{t + d(x, y)}$ for all $x, y \in X$ then $(X, M, *)$ is a fuzzy metric space. Where $a * b =$ min{a, b}.

$$A(x) = \begin{cases} .25, 0 \le x \le .52 \\ \dfrac{x}{2}, x > .52 \end{cases} \qquad S(x) = \begin{cases} .25, 0 \le x \le .6 \\ x - .25, x > .6 \end{cases}$$

$$T(x) = \begin{cases} .25, 0 \le x \le .6 \\ \dfrac{x}{4}, x > .6 \end{cases} \qquad B(x) = \begin{cases} .25, 0 \le x \le .95 \\ x - .75, x > .95 \end{cases}$$

We define $x_n = .5 + \dfrac{1}{n}$ and $y_n = 1 + \dfrac{1}{n}$

We have $A(X) = \{.25\} \cup (.26, 1]$

$S(X) = \{.25\} \cup (.35, 1.75]$

$T(X) = (.15, .5]$ and

$B(X) = \{.25\} \cup (.20, 1.25]$

Also $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} \dfrac{1}{2}[.5 + \dfrac{1}{n}] = .25 \in S(X)$

$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} .5 + \dfrac{1}{n} - .25 = .25 \in A(X)$

$$\lim_{n\to\infty} Ty_n = \lim_{n\to\infty} \frac{1}{4}[1+\frac{1}{n}] = .25 \in B(X) \text{ and}$$
$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} 1+\frac{1}{n} - .75 = .25 \in T(X)$$

R**ole of E.A. property** in proving common fixed point theorems can be concluded by following,

(I) It buys containment of ranges without any continuity requirements.
(II) It minimizes the commutativity conditions of the maps to the commutativity at their points of coincidence.
(III) It allows replacing the completeness requirement of the space with a more natural condition of closeness of the range.

Of course, if two mappings satisfy E.A. like property then they satisfy E.A. property also, but, on the other hand, E.A. like property relaxes the condition of containment of ranges and closeness of the ranges to prove common fixed point theorems, which are necessary with E.A .property

Definition [1.12]: A binary operation $*: [0,1] \times [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if $([0,1],*)$ is an abelian topological monoid with unit 1 such that $a_1 * b_1 * c_1 \le a_2 * b_2 * c_2$ whenever $a_1 \le a_2, b_1 \le b_2, c_1 \le c_2$ for all a_1, a_2, b_1, b_2 and c_1, c_2 are in $[0,1]$.

Definition [1.13]: The 3-tuple $(X, M,*)$ is called a 2-fuzzy metric space if X is an arbitrary set , * is a continuous t-norm and M is a fuzzy set in $X^3 \times [0,\infty)$ satisfying the following conditions : for all $x, y, z, u \in X$ and $t_1, t_2, t_3 > 0$.
$(FM'-1)$ $M(x,y,z,0) = 0$,
$(FM'-2)$ $M(x,y,z,t) = 1$, t > 0 and at least two of the three points are equal ,
$(FM'-3)$ $M(x,y,z,t) = M(x,z,y,t) = M(y,z,x,t)$,
(Symmetry about three variables)
$(FM'-4)$ $M(x,y,z,t_1+t_2+t_3) \ge M(x,y,u,t_1) * M(x,u,z,t_2) * M(u,y,z,t_3)$
(This corresponds to tetrahedron inequality in 2-fuzzy metric space).
The function value $M(x,y,z,t)$ may be interpreted as the probability that the area of triangle is less than t.
$(FM'-5)$ $M(x,y,z,.): [0,1) \to [0,1]$ is left continuous.

Definition[1.14] : Let $(X, M,*)$ be a 2-fuzzy mertic space :
(1) A sequence $\{x_n\}$ in 2-fuzzy metric space X is said to be convergent to a point $x \in X$,if

$$\lim_{n\to\infty} M(x_n, x, a, t) = 1$$

For all $a \in X$ and $t > 0$.

(2) A sequence $\{x_n\}$ in 2-fuzzy metric space X is called a Cauchy sequence , if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, a, t) = 1$$

For all $a \in X$ and $t > 0, p > 0$.

(3) A 2-fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition [1.15]: A pair of mappings A and S is said to be weakly compatible in 2-fuzzy metric space if they commute at coincidence points.

2.Main Result:

In 2010,Sanjay Kumar [18] proved following results using E.A. property:

Theorem2.1: Let $f \,\&\, g$ be self maps of a fuzzy metric space $(X, M, *)$,satisfying $M(x, y, t) > 0$ for all x, y in X and $t > 0$ such that following conditions holds:

(I) $M(fx, fy, t) \geq r(M(gx, gy, t))$, for all x, y in X and $t > 0$, where $r:[0,1] \to [0,1]$ is a continuous function such that $r(t) > t$ for each $0 < t < 1, r(0) = 0 \,\&\, r(1) = 1$

(II) $f \,\&\, g$ satisfy the $E.A$. Property,

(III) $g(X)$ is a closed subspace of X.If there exist sequences $\{x_n\} \,\&\, \{y_n\}$ in X such that $x_n \to x \,\&\, y_n \to y$, and $t > 0$, then $M(x_n, y_n, t) \to M(x, y, t)$.

Then $f \,\&\, g$ have a unique common fixed point in X provided $f \,\&\, g$ are weakly compatible maps.

Now we prove our main result for weakly compatible maps under E.A. like proerty as follows:

Theorem2.2: Let $f \,\&\, g$ be self maps of a 2-fuzzy metric space $(X, M, *)$,satisfying $M(x, y, a, t) > 0$
for all $x, y, a \in X$ and $t > 0$ such that following conditions holds

(I) $M(fx, fy, a, qt) \geq min\{M(fx, gy, a, t), M(gx, fy, a, t), M(gx, gy, a, t)\}$

(II) for $x, y, a \in X$ ..

(III) $f \,\&\, g$ satisfy the E.A.Like Property,

Then $f \,\&\, g$ have a unique common fixed point in X provided $f \,\&\, g$ are weakly compatible maps.

Proof: Since $f \,\&\, g$ satisfy E.A Like property therefore , there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z \in f(X)$ or $g(X)$.

Suppose that $\lim_{n\to\infty} fx_n = z \in g(X)$ therefore $z = gu$ for some $u \in X$.

Now we show that $fu = gu$ .from (I) we have;

$$M(fu, fx_n, a, qt) \geq min\{M(fu, gx_n, a, t), M(gu, fx_n, a, t), M(gu, gx_n, a, t)\}$$

Taking $\lim_{n\to\infty}$ we get;

$$M(fu, z, a, qt) \geq min\{M(fu, z, a, t), M(z, z, a, t), M(z, z, a, t)\}$$

$$M(fu, z, a, qt) \geq M(fu, z, a, t)$$

Which is a contradiction,Hence $fu = z = gu$ i.e. u is coincidence point of f & g .
Since f & g are weakly compatible, therefore $fz = fgu = gfu = gz$.
Now we show that $fz = z$,if not from (I) we have;

$$M(fz, fx_n, a, qt) \geq minc\{M(fz, gx_n, a, t), M(gz, fx_n, a, t), M(gz, gx_n, a, t)\}$$

Taking $\lim_{n\to\infty}$ we get;

$$M(fz, z, a, qt) \geq min\{M(fz, z, a, t), M(gz, z, a, t), M(gz, z, a, t)\}$$
$$M(fz, z, a, qt) \geq min\{M(fz, z, a, t), M(fz, z, a, t), M(fz, z, a, t)\}$$

$$M(fz, z, a, qt) \geq M(fz, z, a, t)$$

Which is a contradiction,hence.
$fz = z = gz$ hence z is a common fixed point of f & g .

Uniqueness: Let z_1 be another fixed point of f & g such that $z_1 \neq z$ then from (I) we have;

$$M(fz, fz_1, a, qt) \geq min\{M(fz, gz_1, a, t), M(gz, fz_1, a, t), M(gz, gz_1, a, t)\}$$

$$M(z, z_1, a, qt) \geq min\{M(z, z_1, a, t), M(z, z_1, a, t), M(z, z_1, a, t)\}$$

$$M(z, z_1, a, qt) \geq M(z, z_1, a, t)$$

which is a contradiction;hence $z = z_1$.

Theorem2.3: Let A, B, S and T be self maps of a fuzzy metric space $(X, M, *)$ satisfying the following conditions:
(I) $A(X) \subset T(X)$ and $B(X) \subset S(X)$,
(II)
$$M(Ax, By, kt) \geq \min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Sx, By, t), M(Ax, Ty, t)\}$$

for all x, y in X and $t > 0$,where $k \in (0,1)$,

(IV) Pairs (A,S) or (B,T) satisfy E.A. property,

(V) Pairs (A,S) and (B,T) are weakly commpatible.

If the range of one of A, B, S and T is a closed subset of X, then A, B, S and T have a common fixed point in X

Theorem2.4:Let A, B, S and T be self maps of a 2-fuzzy metric space $(X, M, *)$ satisfying the following conditions:

(I)

$$M(Ax, By, a, qt) \geq min\{M(Sx, Ty, a, t), M(By, Sx, a, t), M(Ax, Ty, a, t), M(Sx, Ax, a, t)\}$$

for all $x, y, a \in X$ and $t > 0$,where $q \in (0,1)$.

(II) Pairs (A,S) and (B,T) satisfy common E.A.Like property,

(III) Pairs (A,S) and (B,T) are weakly commpatible.

then A, B, S and T have a unique common fixed point in X.

Proof: since (A,S) and (B,T) satisfy common E.A.Like property therefore there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = z$ where $z \in S(X) \cap T(X)$ or $z \in A(X) \cap B(X)$

Suppose that $z \in S(X) \cap T(X)$,now we have

$\lim_{n\to\infty} Ax_n = z \in S(X)$ then $z = Su$ for some $u \in X$.

Now, we claim that $Au = Su$,from (I) we have

$$M(Au, By_n, a, qt) \geq min\{M(Su, Ty_n, a, t), M(By_n, Su, a, t), M(Au, Ty_n, a, t), M(Su, Au, a, t)\}$$

Taking limit $n \to \infty$ we get,

$$M(Au, z, a, qt) \geq min\{M(z, z, a, t), M(z, z, a, t), M(Au, z, a, t), M(z, Au, a, t)\}$$

$$M(Au, z, a, qt) \geq min\{1, 1, M(Au, z, a, t), M(z, Au, a, t)\}$$

$$M(Au, z, a, qt) \geq M(Au, z, a, t)$$

which is a contracdiction, hence $Au = z = Su$.

Again

$\lim_{n\to\infty} By_n = z \in T(X)$ then $z = Tv$ for some $v \in X$

Now, we claim that $Tv = Bv$,from (I) we have

$M(Ax_n, Bv, a, qt)$
$\geq min\{M(Sx_n, Tv, a, t), M(Bv, Sx_n, a, t), M(Ax_n, Tv, a, t), M(Sx_n, Ax_n, a, t)\}$

$M(z, Bv, a, qt) \geq min\{M(z, z, a, t), M(Bv, z, a, t), M(z, z, a, t), M(z, z, a, t)\}$

$M(z, Bv, a, qt) \geq min\{1, M(Bv, z, a, t), 1, 1\}$

$M(z, Bv, a, qt) \geq M(Bv, z, a, t)$

which is a contradiction hence $Bv = z = Tv$

Since the pair (A, S) is weak compatible, therefore $Az = ASu = SAu = Sz$
Now we show that $Az = z$
$M(Az, By_n, a, qt)$
$\geq min\{M(Sz, Ty_n, a, t), M(By_n, Sz, a, t), M(Az, Ty_n, a, t), M(Sz, Az, a, t)\}$

Taking limit $n \to \infty$
$M(Az, z, a, qt) \geq min\{M(Az, z, a, t), M(z, Az, a, t), M(Az, z, a, t), M(Az, Az, a, t)\}$

$M(Az, z, a, qt) \geq min\{M(Az, z, a, t), M(z, Az, a, t), M(Az, z, a, t), 1\}$

$M(Az, z, a, qt) \geq M(Az, z, a, t)$
which is a contradiction.hence $Az = z = Sz$.

The weak compatibility of B and T implies that $Tz = TBv = BTv = Bz$.
$M(Ax_n, Bz, a, qt)$
$\geq min\{M(Sx_n, Tz, a, t), M(Bz, Sx_n, a, t), M(Ax_n, Tz, a, t), M(Sx_n, Ax_n, a, t)\}$

$M(z, Bz, a, qt) \geq min\{M(z, Tz, a, t), M(Bz, z, a, t), M(z, Tz, a, t), M(z, z, a, t)\}$

$M(z, Bz, a, qt) \geq min\{M(z, Bz, a, t), M(Bz, z, a, t), M(z, Bz, a, t), 1\}$

$M(z, Bz, a, qt) \geq M(z, Bz, a, t)$

which is a contradiction.hence $Bz = z = Tz$.
Thus z is common fixed point of $A, B, S \& T$.
Uniqueness:suppose that z_1 is another common fixed point of $A, B, S \& T$ such that $z_1 \neq z$
Then from (I) we have,

$$M(Az, Bz_1, a, qt) \geq min\{M(Sz, Tz_1, a, t), M(Bz_1, Sz, a, t), M(Az, Tz_1, a, t), M(Sz, Az, a, t)\}$$

$$M(z, z_1, a, qt) \geq min\{M(z, z_1, a, t), M(z_1, z, a, t), M(z, z_1, a, t), M(z, z, a, t)\}$$

$$M(z, z_1, a, qt) \geq min\{M(z, z_1, a, t), M(z_1, z, a, t), M(z, z_1, a, t), 1\}$$

$$M(z, z_1, a, qt) \geq M(z, z_1, a, t)$$

which is a contradiction.therefore $z = z_1$.

Next we consider a function
$\varphi: [0,1] \rightarrow [0,1]$
satisfying the conditions

$$\begin{cases} \varphi & \text{If continous and nondecresing on } [0,1] \\ & \varphi(t) > t \; \forall \, t \in (0,1) \end{cases}$$

We note that $\varphi(1) = 1$ for all t in $[0,1]$.
Theorem2.5: Let A, B, S and T be self maps of a 2-fuzzy metric space $(X, M, *)$ with continuous t-norm satisfying the following conditions:

(!)
$$M(Ax, By, a, qt) \geq \varphi\left[min\{M(Sx, Ty, a, t), M(By, Sx, a, t), M(Ax, Ty, a, t), M(Sx, Ax, a, t)\}\right]$$

for all $x, y \in X$, $t > 0$.
(II) Pairs (A, S) and (B, T) satisfy common E.A.Like property,
(I) Pairs (A, S) and (B, T) are weakly commpatible.
then there is a unique common fixed point of A, B, S and T .

The Proof follows from theorem 3.1.

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